

1 Stone Ecliptic

You are given the map of Calakmul, a Maya archaeological site in the Mexican state of Campeche. On the main square (Gran Plaza) there are several buildings; in particular, building VI (Estructura VI) and building IV (Estructura IV). The latter is the complex of three temple towers.

It is known that if one looks from building VI to building IV, on the days of equinoxes and solstices, the rising Sun touches the tops of the respective towers (marked). The latitude of Calakmul is 18° North.

Estimate the height of the towers relative to the observation point. Consider the Sun as a point source.

Please find [calakmul.jpg](#) to zoom in.



2 RR Lyrae

In the field of the globular star cluster M4, observed by the GAIA space telescope, a number of RR Lyrae variables were found. The equatorial coordinates of the center of M4 are $\alpha_0 = 16^h 23^m 35.22^s$, $\delta_0 = -26^\circ 31' 32.7''$. You are given data on the RR Lyrae variables:

- the equatorial coordinates (α, δ) of objects,
- the parallax ϖ in milliarcseconds,
- the components ($\mu_\alpha \cos \delta, \mu_\delta$) of their proper motion,
- the period of pulsation,
- the average magnitudes in G, G_{BP} and G_{RP} bands,
- the average radial velocity $\langle V_r \rangle$,
- the amplitude X_G of pulsation in G band and its error ΔX_G ,
- the absorption A_G in G band.

- Determine which stars do not belong to the cluster.
- Find the period-luminosity relation for RR Lyrae variables of the M4 cluster.
- Estimate the distance to the cluster.
- Estimate the mass of the cluster.

Source ID Name	$\alpha, ^\circ$	$\delta, ^\circ$	ϖ mas	$\mu_\alpha \cos \delta$ mas/yr	μ_δ mas/yr	P days	$\langle G \rangle$ mag	$\langle G_{BP} \rangle$ mag	$\langle G_{RP} \rangle$ mag	$\langle V_r \rangle$ km/s	X_G mag	ΔX_G mag	A_G mag
60...496	245.7225	-27.4102	0.303	-6.089	-2.476	0.767	14.27	14.81	13.55		0.57	0.02	1.21
60...168	245.8310	-26.6659	0.570	-12.015	-19.382	0.507	13.00	13.45	12.43	72.38	0.89	0.07	0.84
...

The full data table can be found in [M4_0.csv](#) file.

3 Statistical Parallax

You are given data on some stars of an open cluster associated with a stellar stream:

- the equatorial coordinates (α, δ) of objects,
- the components $(\mu_\alpha \cos \delta, \mu_\delta)$ of their proper motion,
- the radial velocity V_r .

Consider this cluster to consist only of the specified stars.

- Estimate the coordinates of the apex/radiant for the cluster objects.
- Evaluate the parallaxes of individual objects, plot the distribution of parallaxes.
- Determine the statistical parallax of the cluster.

Name	α	δ	$\mu_\alpha \cos \delta$, mas/yr	μ_δ , mas/yr	V_r , km/s
HD 27990	4 ^h 25 ^m 48 ^s	+18°01'02"	75	−14	40.5
HD 27835	4 ^h 24 ^m 13 ^s	+16°22'44"	90	−22	39.5
...

The full data table can be found in [statistical_parallax.csv](#) file.

4 Nearby Cluster

The data describe one of the nearby galaxy clusters (Eftekhari et al., 2022):

- (α, δ) are the equatorial coordinates of the galaxy,
- R_e is the effective radius of the galaxy,
- $M_{r,e}$ is the absolute magnitude in r band inside the effective radius,
- $\mu_{r,e}$ is the average surface brightness in r band inside the effective radius measured in magnitudes per square arcsecond,
- q is the axis ratio of the projection of galaxy on the plane of the sky.

Determine the following parameters of the galaxy cluster:

- the coordinates of the center of the cluster,
- the constellation where the center of the cluster is located,
- the distance to the cluster,
- the linear dimensions of the cluster in the plane of the sky in α and δ directions.

ID	α , °	δ , °	R_e , "	$M_{r,e}$	$\mu_{r,e}$	q
22_D244	51.6043	−37.1278	8.5	−15.32	21.60	0.64
16_D417	52.9485	−35.0514	19.8	−16.14	22.79	0.76
...

The full data table can be found in [nearby_cluster.csv](#) file.