1 Slow down and shine

Proper motion of some star has reduced by 50 % in 10 000 years. How much has the apparent magnitude of the star changed? Assume that the star was moving linearly at a constant speed.

2 Mysterious Mercury

Here are a few celestial events for October 2022:

October 9 — *Mercury at greatest elongation;*
October 24 — *Mercury occultation by the Moon;*
October 25 — *partial solar eclipse.*

When was the last greatest evening elongation of Mercury before the olympiad? In what constellation was Mercury at that time? Assume Mercury's orbit is circular with radius $r_M = 0.387$ au.

3 Anti-Earth

Imagine that Anti-Earth really exists and moves exactly along the orbit of the Earth so that it passes aphelion at the moment when the Earth passes perihelion and vice versa. Neglect the gravitational interaction between the Earth and Anti-Earth.

Let the Earth be at perihelion, Anti-Earth at aphelion at some point in time. Which of them and how much earlier will cross the minor axis of the orbit? Can observers detect Anti-Earth when the Earth crosses the minor axis of the orbit?

4 Red Skies

A close binary star consists of two components with the same brightness in the V band:

№	Spectral class	True $B-V$	True $U-B$
1	M2	$+1.8$	$+2.1$
2	B8	-0.1	-0.6

Determine the color indices of the binary star after passing through the Earth's atmosphere at an altitude of 45°. Which of the components has a greater color excess due to atmospheric absorption? Consider only Rayleigh scattering $\sigma \propto \lambda^{-4}$, with absorption $A_V = 0.30^{\mathrm{m}}$ in the V band at the zenith.

5 Follow the Shadow

Occultation of a star by the Moon begins on Earth at the North Pole, and ends at a point with latitude 33° North on the prime meridian (0° longitude) at 0^{h} local time. Find the star's equatorial coordinates and the occultation date.

6 Orbital Time Machine

Light-collecting area of the James Webb Space Telescope (JWST) is about 25 m^2. Estimate how long it takes to catch 10 photons emitted by a solar type star in a galaxy at redshift $z = 0.2$. The JWST wavelength coverage is $0.6 - 28.5$ μm (orange to mid-infrared). Assume that the JWST is equipped with a single detector capable of registering photons over the entire wavelength coverage of the telescope.

7 Polar Wi-Fi

A satellite is moving in a polar geocentric orbit with semi-major axis $a = 15\,400$ km, eccentricity $e = 0.55$, and argument of pericenter $\omega = 270°$. Imagine that an observer is at the North Pole of the Earth. The satellite emits a signal with a frequency of 2.4 GHz. At some point in time, the satellite is observed on the horizon.

Estimate

a) the distance from the observer to the satellite,
b) the total velocity of the satellite at that moment,
c) the shift in the observed signal frequency.

8 Perfect Nebula

A planet with synchronous rotation orbits a star in a circular orbit. The planet has no atmosphere, but the system is surrounded by a large homogeneous spherical gaseous nebula. This nebula does not absorb light, but only scatters it isotropically and with the same scattering properties for all wavelengths.

The surface temperature of the planet at the point facing the star is twice that of the opposite point. The Sun's apparent magnitude on the planet's surface is 20^m. Find the distance to the system. Ignore interstellar extinction.

Constants

Universal

Speed of light	$c = 3.00 \cdot 10^8$ m/s
Planck constant	$h = 6.63 \cdot 10^{-34}$ J · s
Hubble constant	$H_0 = 70$ (km/s)/Mpc
Astronomical unit	1 au $= 149.6 \cdot 10^6$ km
Parsec	1 pc $= 206\,265$ au

Earth

Radius	$R_\oplus = 6371$ km
Obliquity	$\varepsilon = 23.4°$
Surface gravity	$g = 9.81$ m/s^2
Orbital period	$T_\oplus = 365.26^d$
Orbital eccentricity	$e_\oplus = 0.0167$

Moon

Radius	$R_☾ = 1737$ km
Orbital period	$T_☾ = 27.32^d$
Orbital inclination	$i_☾ = 5.1°$

Sun

Radius	$R_\odot = 6.96 \cdot 10^5$ km
Absolute magnitude	$M_\odot = 4.74^m$
Effective temperature	$T_\odot = 5.8 \cdot 10^3$ K

UBV system	Mean wavelengths
U band	$\lambda_U = 364$ nm
B band	$\lambda_B = 442$ nm
V band	$\lambda_V = 540$ nm