1 Kholshevnikov Metric

Consider two points in 3-dimensional space, defined by their radius vectors $\vec{r}_1$ and $\vec{r}_2$. According to the Pythagorean theorem, the distance between these points $d_{12} = \sqrt{(\vec{r}_1 - \vec{r}_2)^2}$. The concept of "distance" can be generalized to arbitrary spaces.

In order to determine which meteor shower a meteoroid belongs to, it is required to determine the "distance" between the orbits — the so-called *metric* in the space of orbital parameters. One example of metric is Kholshevnikov metric, described in detail below.

Let a, e, i, ω, Ω be the semi-major axis, eccentricity, inclination, argument of the pericentre and the longitude of the ascending node of the meteoroid's orbit, respectively (Keplerian elements).

We introduce two vectors $\vec{u}$ and $\vec{v}$:

$$\vec{u} := \begin{pmatrix} \sin i\sqrt{a(1-e^2)}\sin\Omega \\ -\sin i\sqrt{a(1-e^2)}\cos\Omega \\ \cos i\sqrt{a(1-e^2)} \end{pmatrix}, \quad \vec{v} := \begin{pmatrix} e\sqrt{a(1-e^2)}(\cos\omega\cos\Omega - \cos i\sin\omega\sin\Omega) \\ e\sqrt{a(1-e^2)}(\cos\omega\sin\Omega + \cos i\sin\omega\cos\Omega) \\ e\sin i\sqrt{a(1-e^2)}\sin\omega \end{pmatrix},$$

Let us define a distance ρ — Kholshevnikov metric — between two orbits in the space of Keplerian elements by the formula

$$\rho = \sqrt{(\vec{u}_1 - \vec{u}_2)^2 + (\vec{v}_1 - \vec{v}_2)^2}.$$

Let a_0, e_0, i_0, ω_0, Ω_0 be the Keplerian elements of the mean orbit of the meteor shower. If the distance ρ between this orbit and the orbit of a given meteoroid is smaller than a limiting value, we may suppose the meteoroid to belong to this meteor shower.

The file `catalog2018.csv` contains the observation times and equatorial coordinates of meteoroids, as well as the Keplerian elements of the orbit of each meteoroid. The data was taken from the archives of the project SonotaCo Network.

Year	Month	Day	α, °	δ, °	a, au	e	i, °	ω, °	Ω, °
2018	1	1.42418	83.56900	1.11400	1.967	0.57950	8.81110	55.40330	100.69660
2018	1	1.48810	107.81900	24.12900	1.973	0.75290	1.44970	280.02610	280.76860
...	...	...	...	...	...	...	...	...	...

We are looking for meteoroids related to the Geminids meteor shower. The mean orbit of the shower corresponds to $a_0 = 1.31$ au, $e_0 = 0.889$, $i_0 = 22.9°$, $\omega_0 = 324.3°$, $\Omega_0 = 261.7°$. For this problem, let us set the limiting value $\log_{10}\rho = -1.0$.

a) Plot all meteoroids on the graph in the coordinates $(\lambda; \beta)$ where λ and β are the ecliptic longitude and latitude of the meteoroid, respectively.

b) How many meteoroids belong to the Geminids meteor shower? Use the limiting value of the $\log_{10}\rho$ specified above and consider only meteoroids with elliptic orbits. Mark these meteoroids on the graph in the coordinates $(\lambda; \beta)$.

c) Plot histograms for semi-major axes and $\log_{10}\rho$ values for meteoroids belonging to the Geminids meteor shower and the histogram for days of observing these meteoroids. On what date was the largest number of meteoroids from the Geminids meteor shower observed?

d) Determine the median ecliptic and equatorial coordinates of the meteoroids belonging to the Geminids meteor shower.

2 Metallica

The file `clusters.csv` contains the parameters of several dozen open clusters of the Milky Way (Viscasillas Vázquez et al., 2023, arXiv:2309.17153). The columns indicate:

- the name of the cluster,
- metallicity [M/H] (dex),
- α-element enhancement [α/Fe] (dex),
- age (in billion years),
- galactocentric distance R_{GC} (in pc),
- orbital eccentricity e,
- maximum deviation from the Galactic plane $Z_{\max}$ (in kpc),
- velocity components in cylindrical galactocentric coordinate system (radial V_R, azimuthal V_ϕ, vertical V_Z, in km/s).

Cluster Name	[M/H] dex	[α/Fe] dex	Age Gyr	R_{GC} pc	e	$Z_{\max}$ kpc	V_R km/s	V_ϕ km/s	V_Z km/s
`Alessi1`	−0.052	−0.029	1.445	8637.1	0.063	0.23	17.29	230.996	12.241
`Berkeley89`	0.047	−0.024	2.089	8473.7	0.173	0.338	28.563	205.102	8.261
...	...	...	...	...	...	...	...	...	...

Here "dex" is a contraction of "decimal exponent", a convenient unit indicating any number or ratio's order-of-magnitude. For example, 100 could be described as 2 dex, or two numbers that differ by a factor of 1000 could be said to differ by 3 dex.

In the cylindrical galactocentric coordinate system,

- the radial velocity V_R is measured along the galactic axial distance,
- the azimuthal velocity V_ϕ is measured in the direction of rotation of the Galaxy,
- the vertical velocity V_Z is measured perpendicular to the galactic disk.

a) What are α-elements? Give three examples.

b) How many times is the ratio of the concentration of metal atoms to the concentration of hydrogen atoms in the oldest cluster of the catalogue greater than in the Sun?

c) Plot a graph of [M/H] versus the α-element enhancement [α/Fe]. Determine the coefficient of linear correlation between these values. Explain the presence or absence of correlation (or anticorrelation). How will the correlation coefficient change when the object with the smallest [α/Fe] is excluded?

The coefficient r_{xy} of linear correlation between the values of x and y is determined by the formula

$$r_{xy} = \frac{\sum_{i=1}^{N} \left(x_i - \langle x \rangle\right)\left(y_i - \langle y \rangle\right)}{\sqrt{\sum_{i=1}^{N} \left(x_i - \langle x \rangle\right)^2} \sqrt{\sum_{i=1}^{N} \left(y_i - \langle y \rangle\right)^2}}\,; \quad \langle x \rangle = \frac{1}{N}\sum_{i=1}^{N} x_i\,, \quad \langle y \rangle = \frac{1}{N}\sum_{i=1}^{N} y_i\,.$$

d) Plot a graph of e versus the ratio V_p/V, where $\vec{V}_p$ is the velocity component perpendicular to $\vec{V}_\phi$ and V is the total velocity. Determine the coefficient of linear correlation between e and V_p/V. Explain the presence or absence of correlation (or anticorrelation).

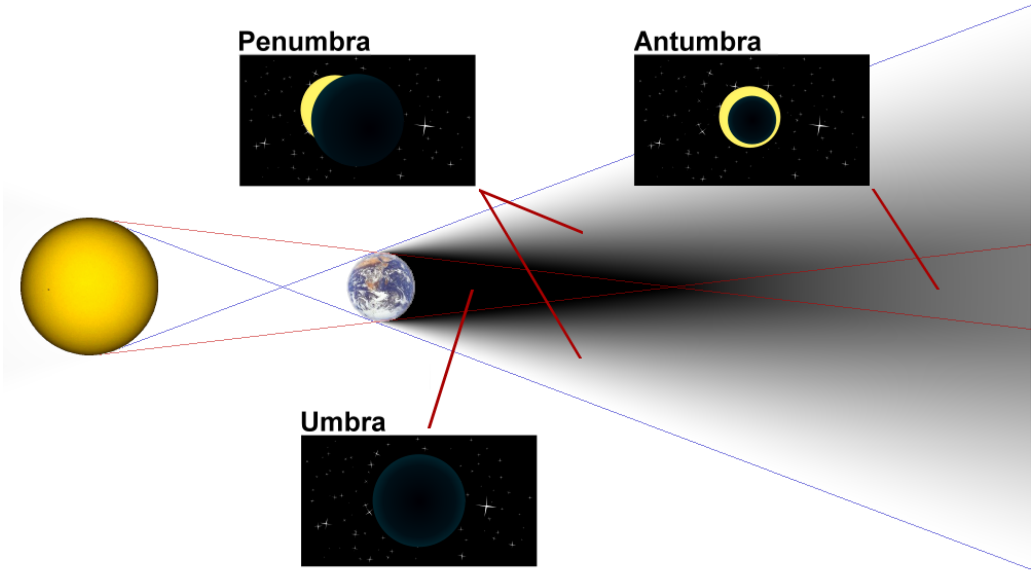
3 Eclipse in Siberia

There are two videos:

- eclipse1.mp4 or `https://youtu.be/nsWVPedsQNk`
- eclipse2.mp4 or `https://youtu.be/Lw7GGm0U94g`

showing conditions for the annular solar eclipse on July 25, 2120. The greatest eclipse will be observed in Siberia, geographical region in Russia. Estimate the altitude of the Sun at the point where the maximum magnitude of the eclipse will be observed (if the weather is good).

Hint: the center of lunar antumbra reaching the Earth's surface is marked by green cross.



Constants

Universal

Speed of light	$c = 3.00 \cdot 10^8$ m/s
Planck constant	$h = 6.63 \cdot 10^{-34}$ J · s
Hubble constant	$H_0 = 70$ (km/s)/Mpc
Astronomical unit	1 au = $149.6 \cdot 10^6$ km
Parsec	1 pc = 206 265 au

Earth

Radius	$R_\oplus = 6371$ km
Obliquity	$\varepsilon = 23.4^\circ$
Surface gravity	$g = 9.81$ m/s^2
Orbital period	$T_\oplus = 365.26^\mathrm{d}$
Orbital eccentricity	$e_\oplus = 0.0167$

Moon

Radius	$R_☾ = 1737$ km
Orbital period	$T_☾ = 27.32^\mathrm{d}$
Orbital inclination	$i_☾ = 5.1^\circ$

Sun

Radius	$R_\odot = 6.96 \cdot 10^5$ km
Absolute magnitude	$M_\odot = 4.74^\mathrm{m}$ (bol.)
Effective temperature	$T_\odot = 5.8 \cdot 10^3$ K
Luminosity	$L_\odot = 3.828 \cdot 10^{26}$ W

Emission constants

Stefan–Boltzmann	$\sigma = 5.67 \cdot 10^{-8}$ (W/m^2)/K^4
Wien's displacement	$b = 2898$ µm · K

UBV system

	Mean wavelengths
U band	$\lambda_U = 364$ nm
B band	$\lambda_B = 442$ nm
V band	$\lambda_V = 540$ nm

Hydrogen spectrum

Lyman Lα	$\lambda_{L\alpha} = 1215.7$ Å
Balmer Hα	$\lambda_{H\alpha} = 6562.8$ Å