1 Ceres

A spacecraft is in a circular orbit with radius $r = 1.3$ au in the ecliptic plane. An additional speed of 5 km/s is instantly given to the spacecraft in the direction of movement, so its orbit intersects with the orbit of 1 Ceres. What will be the relative velocity of the spacecraft and Ceres when the spacecraft crosses its orbit for the first time? The distance between Ceres and the spacecraft at that time is 10^6 km. Assume the orbit of Ceres to be circular with radius $r_C = 2.8$ au. The directions of orbital motion of the objects are the same.

2 Straight Forward

Currently, a star has a proper motion of $0.5''$/year with a parallax of $0.08''$. The hydrogen Hα line in the stellar spectrum is observed at wavelength $\lambda = 6561.0$ Å. Assume the star's velocity vector to be constant. Estimate the radial velocity of the star in 20 000 years.

3 Apparently Invisible

A planet orbits a main sequence star with apparent bolometric magnitude $m = +7^{\mathrm{m}}$ in a circular orbit with period $T = 570$ years. The star is 385 light years from the Sun.

a) Estimate the maximum angular distance between the planet and the star for an observer on the Earth.

b) Compare the obtained value with the angular resolution of the James Webb Space Telescope (JWST) at wavelength $\lambda = 3.8$ µm.

The effective diameter of JWST is $D = 6.5$ m. Neglect the interstellar extinction.

4 Hot Potato

A main sequence star has radius $R = 3.9R_\odot$, effective temperature $T = 9520$ K, and parallax $\varpi = 0.011''$. An asteroid orbits the star with an orbital period of 1 year. The asteroid rotates quite rapidly. The asteroid's surface reflects $A = 30\,\%$ of incident radiation.

Estimate:

a) the average density of the star,

b) the orbital radius of the asteroid,

c) the effective surface temperature of the asteroid.

d) Is it possible to observe this star from the Earth with the naked eye?
The bolometric correction at a given temperature is approximately -0.15.

5 Hide and Seek

At the 50th parallel (50° N), some star rises just before Antares ($\alpha_A = 16^{\mathrm{h}}\,29^{\mathrm{m}}$; $\delta_A = -26^\circ\,26'$) disappears behind the horizon, and it sets just when Sirius ($\alpha_S = 6^{\mathrm{h}}\,45^{\mathrm{m}}$; $\delta_S = -16^\circ\,43'$) appears. The star is brighter than $+1.5^{\mathrm{m}}$. Estimate the equatorial coordinates of the star. What is the name of this star? Consider the Earth to be perfectly spherical with no atmosphere.

6 Space Vodka

The table below shows the characteristics of two space masers.
Which source is larger in size and how many times larger?

	Water H_2O	Methanol CH_3OH
Wavelength, cm	1.35	4.5
Flux density, kJy	1500	0.15
Brightness temperature, $\log T$ [K]	17	6.5
Parallax, mas	2.5	0.75

7 One Thousand and One Nights

On a rocky planet in a circular orbit around a star, equatorial and ecliptic coordinates are arranged in the same way as on the Earth. The local ecliptic passes through points with equatorial coordinates ($\alpha_1 = 20.4°$; $\delta_1 = 22.5°$) and ($\alpha_2 = 74.7°$; $\delta_2 = 49.0°$). Calculate the fraction of the planet's surface where polar nights can occur. Neglect the atmosphere.

8 Omega Sirius

Estimate the distance between α and ω Canis Majoris in parsecs.

	α CMa	ω CMa
Right ascension	$06^h\,45^m\,08.917^s$	$07^h\,14^m\,48.653^s$
Declination	$-16°\,42'\,58.02''$	$-26°\,46'\,21.61''$
Apparent magnitude in V band	-1.46	3.82
Spectral type	A1Vm	B2.5Ve
Bolometric correction	0	-2.2
Mass, $\mathfrak{M}_\odot$	2.063 ± 0.023	10.1 ± 0.7

9 Get Into the Loop

An artificial satellite moves in an orbit with eccentricity $e > 0$, semi-major axis a, and inclination $0 < i < 90°$. The argument of perigee $\omega = 0°$. Assume the Earth to be an ideal sphere rotating at a constant angular velocity W.

a) What are the satellite's geographic latitude $\varphi(\nu)$ and longitude $\lambda(\nu)$ depending on the true anomaly ν?

b) Consider a satellite in a geosynchronous orbit ($T = 23^h\,56^m\,04^s$) with $e = 0.30$ and $i = 1.00$ rad. Calculate the ground track of the satellite (projection of the trajectory onto the surface of the rotating Earth) and draw it. For convenience, use the answer sheet with table and graph grid.

Hint: $$\int \frac{dx}{(1 + a\cos x)^2} = \frac{2\arctan\left(\sqrt{\frac{1-a}{1+a}} \cdot \tan\frac{x}{2}\right)}{(1-a^2)^{3/2}} - \frac{a\sin x}{(1-a^2)(1+a\cos x)} + \text{const.}$$

Constants

Universal		**Sun**	
Speed of light	$c = 3.00 \cdot 10^8$ m/s	Radius	$R_\odot = 6.96 \cdot 10^5$ km
Planck constant	$h = 6.63 \cdot 10^{-34}$ J · s	Absolute magnitude	$M_\odot = 4.74^\mathrm{m}$ (bol.)
Hubble constant	$H_0 = 70$ (km/s)/Mpc	Effective temperature	$T_\odot = 5.8 \cdot 10^3$ K
Astronomical unit	1 au = $149.6 \cdot 10^6$ km	Luminosity	$L_\odot = 3.828 \cdot 10^{26}$ W
Parsec	1 pc = 206 265 au	**Emission constants**	
Earth		Stefan–Boltzmann	$\sigma = 5.67 \cdot 10^{-8}$ (W/m^2)/K^4
Radius	$R_\oplus = 6371$ km	Wien's displacement	$b = 2898$ µm · K
Obliquity	$\varepsilon = 23.4^\circ$	**UBV system**	Mean wavelengths
Surface gravity	$g = 9.81$ m/s^2	U band	$\lambda_U = 364$ nm
Orbital period	$T_\oplus = 365.26^\mathrm{d}$	B band	$\lambda_B = 442$ nm
Orbital eccentricity	$e_\oplus = 0.0167$	V band	$\lambda_V = 540$ nm
Moon		**Hydrogen spectrum**	
Radius	$R_☾ = 1737$ km	Lyman Lα	$\lambda_{L\alpha} = 1215.7$ Å
Orbital period	$T_☾ = 27.32^\mathrm{d}$	Balmer Hα	$\lambda_{H\alpha} = 6562.8$ Å
Orbital inclination	$i_☾ = 5.1^\circ$		