1 Crossroad of Worlds

The table below shows the current equatorial coordinates (α, δ), parallax ϖ, proper motion components (μ_α, μ_δ), and radial velocity V_r for two stars for the observer on the Earth. Consider the motion of the stars relative to the Sun to be linear. Determine when the angular distance between the stars will reduce by 50 % as observed from the Earth.

Star	(α, δ)	ϖ (mas)	μ_α (mas/yr)	μ_δ (mas/yr)	V_r (km/s)
A	$(10^\mathrm{h}\,48^\mathrm{m}\,00^s,\ 0^\circ\,0'\,0'')$	80	-160	0	-25
B	$(10^\mathrm{h}\,00^\mathrm{m}\,00^\mathrm{s},\ 0^\circ\,0'\,0'')$	60	$+180$	0	-20

2 Satellite Tracking

A satellite is moving in a circular orbit around the Earth with an inclination of 20° and an orbital radius of 30 000 km. At noon, the satellite passes through the zenith at a point P_1 with coordinates $(0^\circ\,\mathrm{N},\ 10^\circ\,\mathrm{E})$.

a) At which point P_2 of the Earth will the satellite pass through the zenith after 3 hours?

b) At what astronomical altitude will it be for the observer at P_1 while passing over P_2?

3 Planet on the Spot

There is a large round starspot with a temperature of 2900 K near the center of the disk of a far away star with a radius of $0.7R_\odot$ and a temperature of 3400 K. The decrease in brightness due to the presence of the spot turns out to be the same as during the central transit of a planet with a radius of $6 \cdot 10^4$ km. The axis of rotation of the star is perpendicular to the line of sight, the rotation period is 20 earth days.

a) Estimate the radius of the spot.

b) Estimate the duration of the central transit of the planet through the spot if the radius of the planet's orbit is 0.6 au. The planet's orbit lies in the plane of the star's equator.

4 Let the Sky Fall

When the sun rises in the west and sets in the east, when the seas go dry and mountains blow in the wind like leaves... (George R. R. Martin)

a) What should be the minimum eccentricity of the Earth's orbit so that the Sun sometimes rises in the west? Consider the semi-major axis of the Earth's orbit and the sidereal day to remain the same as today. Neglect the obliquity of the ecliptic.

b) Estimate the maximum surface temperature of the Earth when the Sun rises in the west. Consider the spherical albedo of the Earth to be equal to 0.3.

c) Can the Earth have an atmosphere under such conditions? Prove your answer with calculations.

5 Almost Circumpolar

Let's consider stars for which it is true at the same time: (i) in their diurnal motion they cross the horizon, and (ii) their upper culmination occurs to the north of the zenith.

a) Express the fraction of such stars as a function of latitude φ. Assume that stars are distributed uniformly over the celestial sphere.

b) Determine what is the maximum possible fraction of such stars and at what latitude this maximum is achieved.

6 Jupiter in Sirius

Let's observe Jupiter from a point near the Omega Sirius Hotel tonight!

a) In which constellation is Jupiter located?

b) At what civil time and at what altitude the upper culmination of Jupiter does occur? Neglect the equation of time.

c) Estimate the period of visibility of Jupiter during the current day.

d) What is the elongation of Jupiter?

e) What is the angular size of Jupiter?

f) Estimate the date of the nearest opposition of Jupiter in the future.

Date		16 September 2024
Jupiter	Right ascension α	$5^{\mathrm{h}}\,17^{\mathrm{m}}\,28^{\mathrm{s}}$
	Declination δ	$+22^\circ\,22.0'$
Omega Sirius	Latitude φ	$43^\circ\,24.7'$ N
	Longitude λ	$39^\circ\,57.0'$ E
	Time zone	Moscow (UT+3)

7 Yet Another Pinwheel

The radial surface-brightness profile of disc galaxies is overall well described by an exponential function:

$$I(R) = I_0 \exp\left(-\frac{R}{h}\right),$$

where R is the distance from the center of the galaxy, h is the radial scale length and I_0 is the central surface brightness measured in $L_\odot/\mathrm{pc}^2$.

In the spectrum of the observed face-on disc galaxy (the axis of the galaxy coincides with the line of sight), the center of the H_α line is observed at a wavelength of 6670 Å. The apparent magnitude of the galaxy is 14^{m} and h is assumed to be 3 kpc.

a) Estimate the effective radius R_e of the galaxy. R_e is the radius within which 50 % of the total light of a galaxy is emitted.

b) Estimate the central surface brightness I_0.

c) Estimate the baryonic mass of the galaxy.

8 Thirty Degrees of Freedom

A spherical molecular cloud has a radius of 5 pc and a mass of $2 \cdot 10^3 \mathfrak{M}_\odot$. The temperature of the gas is 30 K, the cloud is homogeneous.

a) Estimate the mean free path and the mean free time (between successive collisions) of the hydrogen molecules in the cloud. The cross section radius of H_2 is 2.7 Å.

b) Let a star pass behind the cloud along its apparent diameter with a tangential velocity of 70 km/s. For how long will the absorption be greater than 0.5^{m}? Absorption in the V band is related to the atomic column density by formula $A_V = 5.2 \cdot 10^{-22}\ N_H$ in magnitudes, where N_H is measured in cm^{-2}.

9 High Precision Guidance

A radiotelescope RT-22 (CrAO) has a mirror dish with a diameter of 22 m and focal length $F = 9.5$ m. They are going to observe the Sun in the first sidelobe of the radiation pattern of the telescope (see Figure 1) at a frequency of 420 MHz. A receiver is located at the primary (main) focus of the mirror. Consider the mirror to be spherical. Determine the distance between the center of the mirror and the center of the receiver's shadow on the dish.

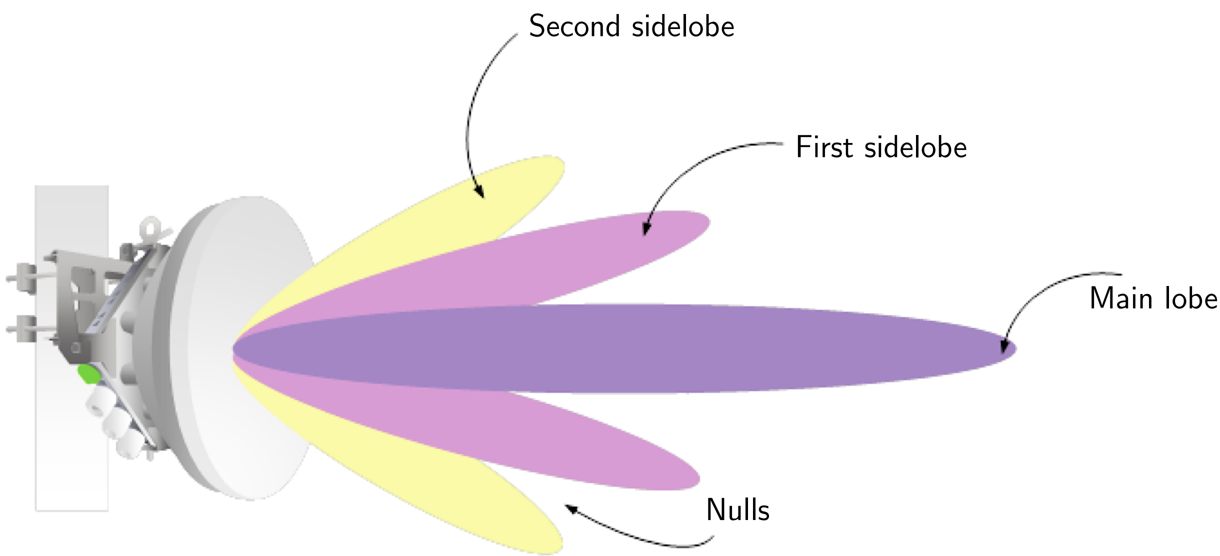

Figure 1: Radiation pattern lobes (beams) of an antenna

Constants

Universal

Speed of light	$c = 3.00 \cdot 10^8$ m/s
Planck constant	$h = 6.63 \cdot 10^{-34}$ J · s
Boltzmann constant	$k_B = 1.38 \cdot 10^{-23}$ J/K
Gas constant	$\mathfrak{R} = 8.314$ J/(mol · K)
Proton mass	$m_p = 1.673 \cdot 10^{-27}$ kg

Astronomical

Astronomical unit	1 au = $149.6 \cdot 10^6$ km
Parsec	1 pc = 206 265 au
Hubble constant	$H_0 = 70$ (km/s)/Mpc

Earth

Radius	$R_\oplus = 6371$ km
Mass	$\mathfrak{M}_\oplus = 5.97 \cdot 10^{24}$ kg
Obliquity of ecliptic	$\varepsilon = 23.4^\circ$
Surface gravity	$g = 9.81$ m/s^2
Orbital period	$T_\oplus = 365.26^\mathrm{d}$
Orbital eccentricity	$e_\oplus = 0.0167$

Hydrogen spectrum

Lyman Lα	$\lambda_{\mathrm{L}\alpha} = 1215.7$ Å
Balmer Hα	$\lambda_{\mathrm{H}\alpha} = 6562.8$ Å

Jupiter

Radius	$R_J = 6.99 \cdot 10^4$ km
Mass	$\mathfrak{M}_J = 1.90 \cdot 10^{27}$ kg
Orbital radius	$a_J = 5.20$ au
Orbital period	$T_J = 11.86$ yr

Sun

Radius	$R_\odot = 6.96 \cdot 10^5$ km
Mass	$\mathfrak{M}_\odot = 1.99 \cdot 10^{30}$ kg
Absolute magnitude	$M_\odot = 4.74^\mathrm{m}$ (bol.)
Effective temperature	$T_\odot = 5.8 \cdot 10^3$ K
Luminosity	$L_\odot = 3.828 \cdot 10^{26}$ W

Emission constants

Stefan–Boltzmann	$\sigma = 5.67 \cdot 10^{-8}$ (W/m^2)/K^4
Wien's displacement	$b = 2898$ µm · K

UBV system	Mean wavelengths
U band	$\lambda_U = 364$ nm
B band	$\lambda_B = 442$ nm
V band	$\lambda_V = 540$ nm

Number of Siriuses in the Solar system $\geqslant 1$