1 Chandrasekhar Says No

The file WD.txt contains a data table for ZZ Ceti white dwarfs (Cang et al., 2025) with the following columns:

- `Teff(K)` — effective temperature in kelvin;
- `logg` — base-10 logarithm of the surface gravity in cm/s^2;
- `G(mag)` — apparent magnitude in the G band;
- `Parallax(mas)` — parallax in milliarcseconds.

The file BC.txt provides bolometric corrections for the G band (Carrasco et al., 2014):

- `Teff[K]` — effective temperature in kelvin;
- `logg[cm/s2]` — base-10 logarithm of the surface gravity in cm/s^2;
- `Mbol-MG` — bolometric correction.

a) Plot the dependence of the white dwarf radius (in $R_\odot$) on its mass (in $\mathfrak{M}_\odot$).

b) How many objects outlie the main relation? For how many objects are the parameters non-realistic? Explain.

2 At the World's Edge

Cape Fligely (81° 51′ N; 59° 14′ E) is located on the northern coast of Rudolf Island in Franz Josef Land, Russian Federation. It is the northernmost point of Russia, Europe, and the Eurasian continent as a whole.

a) Find the distance from the cape to the North Pole of the Earth.

b) At what solar declination does the polar day begin at the cape? Assume that atmospheric refraction at the horizon is 35′.

You are provided with a table eclipses.xlsx of lunar eclipses of the 21st century. The table lists the eclipse date, Saros number, gamma[1], the moments of the Moon's disk contacts with the Earth's shadow (see Fig. 1), and the time of greatest eclipse.

c) How many TOTAL lunar eclipses will occur in the 21st century? PARTIAL? PENUMBRAL?

Point Ξ (0; 59° 14′ E) lies on the equator at the same longitude, somewhere in the Indian Ocean.

d) Determine how many TOTAL lunar eclipses will be visible from both Cape Fligely and point Ξ. List the corresponding event numbers.

Count an eclipse as visible if any part of totality occurs above the horizon. Ignore events visible only in partial phases.

[1] Gamma is the distance of the center of the Moon's disk from the center of the Earth's umbral shadow, expressed in units of Earth's equatorial radius. It is defined at the moment of greatest eclipse, when its absolute value reaches a minimum. For a lunar eclipse, it shows whether the Moon passes north or south of the center of the Earth's shadow; a positive value means the Moon passes to the north of the center.

3 Beyond the World's Edge

Fractal dimension (D) is a fractional value that measures the complexity and roughness of a shape. It describes how a detail-rich structure fills space and often exhibits self-similarity — meaning parts of the structure resemble the whole, at least statistically, when scaled. For example, a perfectly random distribution of points in a plane would have a fractal dimension equal to the dimension of the space itself, $D = 2$. A value less than 2 indicates a "clumpy" structure with significant empty spaces (voids). The transition scale from a fractal to a homogeneous universe is an active area of research.

The file `SN_cat.csv` provides observational data for supernova events. The columns are:

- `sn_name` supernova designation;
- `redshift` measured redshift;
- `gal_type` morphological type of the host galaxy;
- `sn_ra` right ascension in hours, minutes, and seconds;
- `sn_dec` declination in degrees, minutes, and seconds.

a) Plot the objects on a rectangular sky map with axes (Right Ascension; Declination) in degrees. Are there any regions without objects? What does the largest such region represent?

For the remainder of the problem, ***we will only consider a region of the sky***

$$\delta \in [-5^\circ;\ +5^\circ],$$
$$\alpha \in [300^\circ;\ 360^\circ] \cup [0^\circ;\ 60^\circ],$$

where the object sample seems to be complete.

b) For the selected region, plot a histogram of the redshift distribution.

Keep only objects with redshift $z \leqslant 0.1$.
For these objects, we will determine the fractal dimension. To do this, it is necessary to calculate the pairwise distances $l_{i,j}$ for all pairs (i, j) of the remaining objects. The redshift is small — use ordinary Euclidean distance without applying cosmological models.

c) Plot a histogram for the distribution $f(l)$ of pairwise distance values.

d) To determine the fractal dimension, use pairwise distances l in the range from 1 Mpc to 100 Mpc. Fit the distribution with a power law $f(l) = A\, l^D$.

The sample objects populate an oblate, practically 2D-region. Therefore, D is the fractal dimension of the two-dimensional projection. We may use Mandelbrot's empirical rule to estimate the fractal dimension D_0 of the spatial (3D) distribution of supernovae: $D_0 = D + 1$.

e) Write down the obtained value of D_0. Is D_0 greater or less than 3?

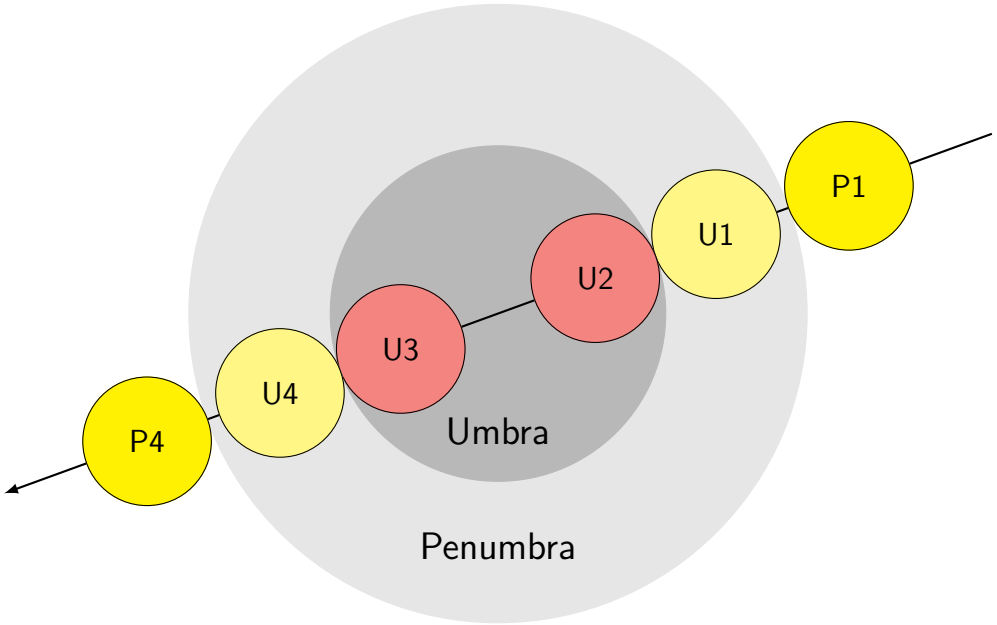

Figure 1: Designation of contacts illustrated with the example of a central total lunar eclipse

Constants

Universal

Gravitational constant	G	$= 6.67 \cdot 10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg \cdot s}^2}$
Speed of light	c	$= 3.00 \cdot 10^8$ m/s

Astronomical

Astronomical unit	1 au	$= 149.6 \cdot 10^6$ km
Parsec	1 pc	$= 206\,265$ au
Hubble constant	H_0	$= 70$ (km/s)/Mpc

Emission constants

Stefan–Boltzmann	σ	$= 5.67 \cdot 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4}$
Wien's displacement	b	$= 2898\ \mu\mathrm{m} \cdot \mathrm{K}$

Earth

Radius	$R_\oplus$	$= 6371$ km
Obliquity of ecliptic	ε	$= 23.4°$
Orbital period	$T_\oplus$	$= 365.26$ days

Sun

Radius	$R_\odot$	$= 6.96 \cdot 10^5$ km
Mass	$\mathfrak{M}_\odot$	$= 1.99 \cdot 10^{30}$ kg
Absolute magnitude	$M_\odot$	$= 4.74^{\mathrm{m}}$ (bol.)
Effective temperature	$T_\odot$	$= 5.8 \cdot 10^3$ K
Luminosity	$L_\odot$	$= 3.828 \cdot 10^{26}$ W