1 Celestial Pause

From opposition to the nearest stationary point, the apparent angular diameter of an asteroid decreases by 10 %. A stationary point is the point at which the asteroid's apparent motion reverses. The observer is located on the surface of the Earth. Assume the asteroid's orbit is circular and lies in the ecliptic plane.

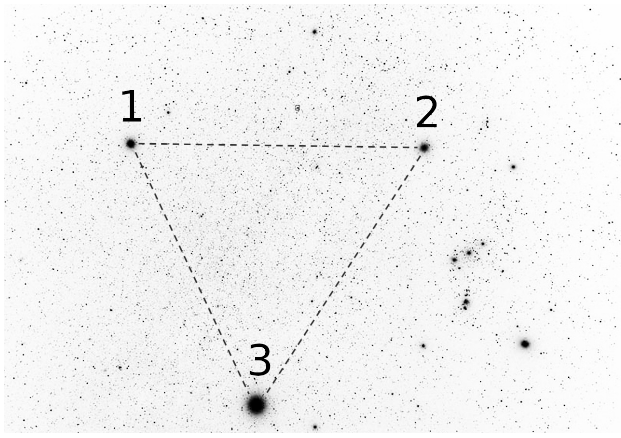
a) Determine the radius of the asteroid's orbit.

b) How much time passes between these two moments?

2 Winter Ensemble

The Winter Triangle is an asterism formed by Sirius, Procyon, and Betelgeuse. Their equatorial coordinates are listed in the table.

Star	Right Ascension	Declination
Sirius	$6^h\,45^m$	$-16°\,45'$
Procyon	$7^h\,40^m$	$+05°\,10'$
Betelgeuse	$5^h\,56^m$	$+07°\,24'$



a) Write down which star is labeled by each number.

b) On what fraction of the Earth's surface are all three stars of the Winter Triangle ALWAYS above the horizon?

c) On what fraction of the Earth's surface can all three stars be above the horizon AT THE SAME TIME at least sometimes?

d) At what latitude can all three stars be at the same altitude AT THE SAME TIME?

Neglect atmospheric refraction.

3 Swing of the Spheres

Because of annual aberration, a G2V star (Sun-like) draws an ellipse on the celestial sphere with an eccentricity of 0.4. Its annual parallax is 1 % of the maximum aberrational shift.

a) Determine the distance to the star.

b) Estimate the minimum baseline of an optical interferometer that can resolve at least some details on the stellar disk at such distance.

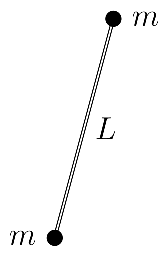
c) What is the apparent magnitude of such an object? Are there any known G2V stars at such distance?

d) Estimate the surface brightness of the stellar disk. Express the result in mag/arcsec^2.

e) Find the possible altitude range of the upper culmination of this star for an observer in the city of Sochi (43.6° N).

4 Tides on a Rod

An experimental gradiometer satellite consists of a thin, rigid, massless rod of length $L = 40$ m with small masses $m = 500$ kg at its ends. Its center moves in a circular orbit at an altitude $h = 400$ km above the Earth's surface. In equilibrium, the rod is oriented radially toward the Earth's center.



Assume that the Earth's gravitational field is the same as that of a point mass:

$$\mathbf{g}(\mathbf{r}) = -\frac{G\mathfrak{M}_\oplus}{r^3}\mathbf{r}.$$

a) Determine the orbital period of the satellite, neglecting the rod's length.

b) Find a small correction to the satellite's circular orbital speed that comes from the fact that it isn't a point, but has a finite length L:

$$v \approx \sqrt{\frac{G\mathfrak{M}_\oplus}{R_\oplus + h}}\left(1 + [\text{SOME NUMBER}]\frac{L^2}{(R_\oplus + h)^2}\right).$$

c) Find the period of small oscillations of the satellite about the equilibrium in the orbital plane, assuming that the satellite's center continues to move in a circular orbit.

5 Above the Photosphere

For a main-sequence star, the difference in free-fall acceleration in the photosphere and at the upper boundary of the chromosphere (10 000 km above the photosphere), measured above the star's pole, is 8 m/s^2. It is also known that from the upper boundary of the chromosphere, only 0.7 % of the entire photosphere is visible.

a) Estimate the mass, radius, and temperature of the star.

b) Which spectral and luminosity class does this star belong to?

c) Estimate the minimum possible rotational period of the star.

6 Glimpse of GLIMPSE

The cluster GLIMPSE-C01 is located in the Milky Way. The tip of the red giant branch (TRGB) corresponds to an apparent magnitude of $K = 8.7^\mathrm{m}$. Its absolute magnitude, assuming a metallicity of [Fe/H] = −1.5, is $M_K = -6.1^\mathrm{m}$ (Ivanov et al., 2005).

a) Assuming the extinction is $a_K = 0.45$ mag/kpc, estimate the distance to the cluster.

b) Estimate the total extinction A_V in the V band, given the cluster's Galactic coordinates: latitude $b = 0^\circ$, longitude $l = 31^\circ$.
NOTE. According to Rieke & Lebofsky (1985), $a_V/a_K = 9$.

c) In reality, the extinction, even in the K band, is uncertain: $a_K = 0.45 \pm 0.08$ mag/kpc. Find the uncertainty in the distance.

d) Is this cluster more likely a globular cluster or an open cluster? Explain.

7 Nothing to Do There

A spacecraft moves along a Hohmann transfer ellipse in the ecliptic plane from the Earth to Jupiter. This ellipse touches the Earth's orbit at perihelion and Jupiter's orbit at aphelion.

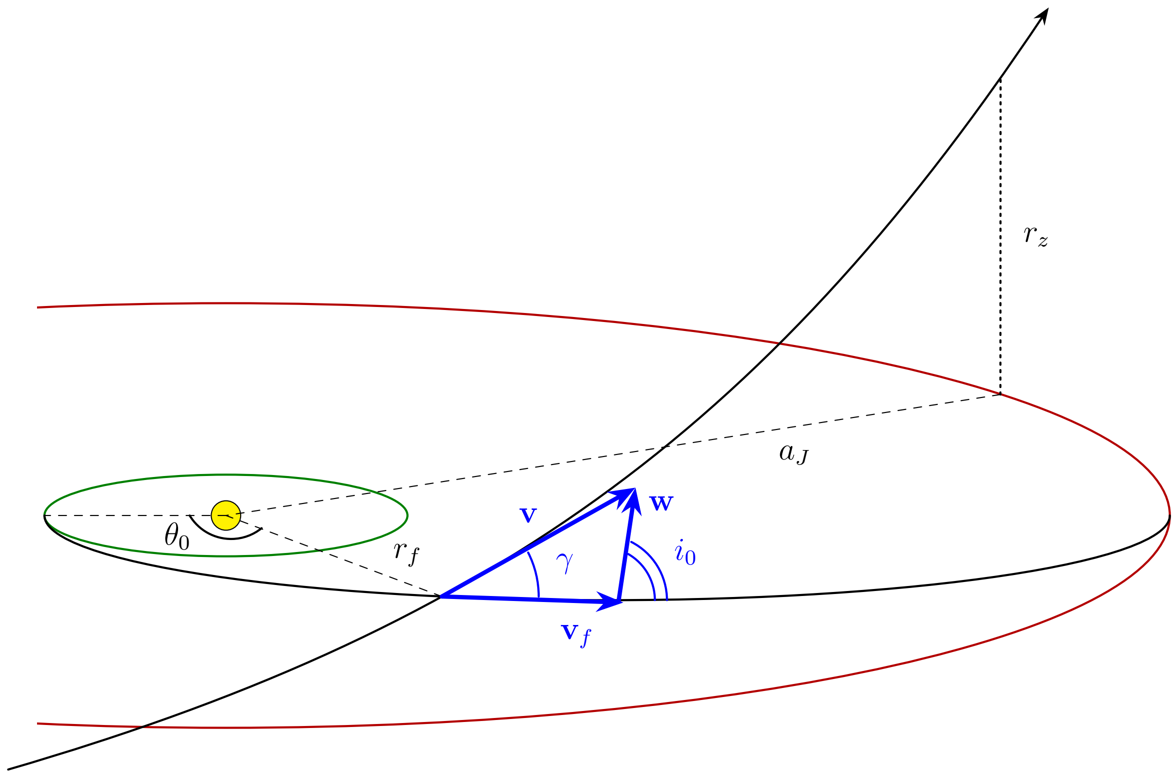
a) How long would the spacecraft take to travel from the Earth to Jupiter along this trajectory?

At true anomaly $\theta_0 = 90^\circ$ (heliocentric distance r_f), the engine is fired briefly. The spacecraft receives a velocity change $\mathbf{w}$ with magnitude $|\mathbf{w}| = \frac{1}{2}|\mathbf{v}_f|$, where $\mathbf{v}_f$ is its current velocity. The impulse makes an angle $i_0 = 20^\circ$ to the plane of the initial orbit, and its projection onto the ecliptic is aligned with the initial velocity. Assume the Earth's and Jupiter's orbits are circular and lie in the same plane.

b) By what angle γ does the spacecraft's velocity vector turn after the impulse?

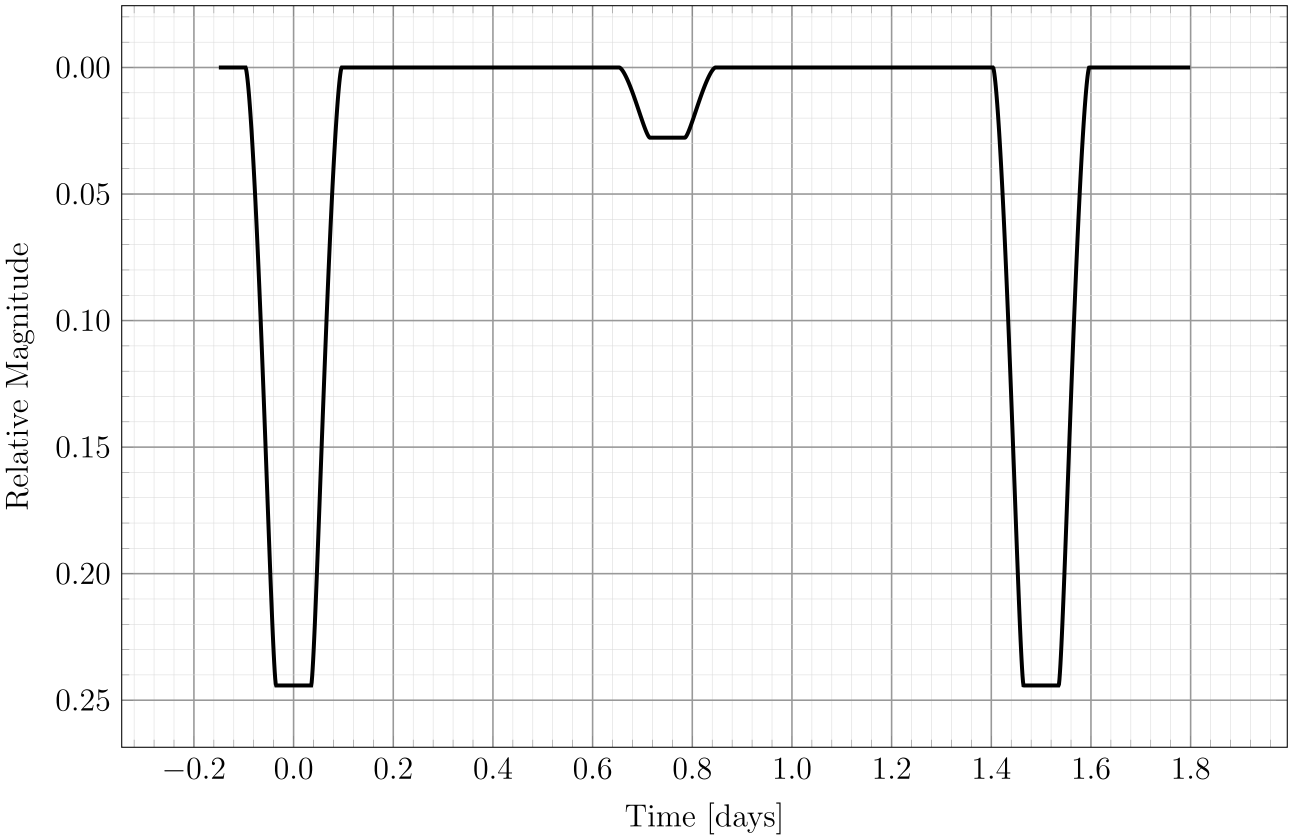
c) Find the semi-major axis, eccentricity, and inclination of the new orbit after the impulse.

d) How far above Jupiter's orbit (r_z) will the spacecraft fly by?



8 Stellar Blends

The figure shows the light curve of an eclipsing binary system observed in the V band. The eclipses in the system are central. Both components are main-sequence stars, and one has spectral type A0 V.



a) Determine the spectral type of the second component of the system.

b) Estimate the distance between the two components of the system.

c) What is wrong with the given light curve?

d) Sketch the curve of the system's color index $(B - V)$ as a function of orbital phase, still assuming the validity of the given light curve.

Constants

Universal

Gravitational constant	$G = 6.67 \cdot 10^{-11}\ \frac{\mathrm{m^3}}{\mathrm{kg \cdot s^2}}$
Speed of light	$c = 3.00 \cdot 10^{8}$ m/s
Planck constant	$h = 6.63 \cdot 10^{-34}$ J · s
Boltzmann constant	$k_B = 1.38 \cdot 10^{-23}$ J/K
Gas constant	$\mathfrak{R} = 8.314$ J/(mol · K)
Proton mass	$m_p = 1.673 \cdot 10^{-27}$ kg

Astronomical

Astronomical unit	1 au = $149.6 \cdot 10^{6}$ km
Parsec	1 pc = 206 265 au
Hubble constant	$H_0 = 70$ (km/s)/Mpc

Earth

Radius	$R_\oplus = 6371$ km
Mass	$\mathfrak{M}_\oplus = 5.97 \cdot 10^{24}$ kg
Obliquity of ecliptic	$\varepsilon = 23.4^\circ$
Surface gravity	$g = 9.81$ m/s^2
Orbital period	$T_\oplus = 365.26$ days
Orbital eccentricity	$e_\oplus = 0.0167$

Hydrogen spectrum

Lyman Lα	$\lambda_{\mathrm{L}\alpha} = 1215.7$ Å
Balmer Hα	$\lambda_{\mathrm{H}\alpha} = 6562.8$ Å

Jupiter

Radius	$R_J = 6.99 \cdot 10^{4}$ km
Mass	$\mathfrak{M}_J = 1.90 \cdot 10^{27}$ kg
Orbital radius	$a_J = 5.20$ au
Orbital period	$T_J = 11.86$ yr

Sun

Radius	$R_\odot = 6.96 \cdot 10^{5}$ km
Mass	$\mathfrak{M}_\odot = 1.99 \cdot 10^{30}$ kg
Absolute magnitude	$M_\odot = 4.74^{\mathrm{m}}$ (bol.)
Effective temperature	$T_\odot = 5.8 \cdot 10^{3}$ K
Luminosity	$L_\odot = 3.828 \cdot 10^{26}$ W
Color index	$(B - V)_\odot = +0.65^{\mathrm{m}}$

Emission constants

Stefan–Boltzmann	$\sigma = 5.67 \cdot 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m^2 \cdot K^4}}$
Wien's displacement	$b = 2898$ µm · K

UBV... system	Mean wavelengths
U band	$\lambda_U = 364$ nm
B band	$\lambda_B = 442$ nm
V band	$\lambda_V = 540$ nm
K band	$\lambda_K = 2190$ nm