1 Chandrasekhar Says No

The file WD.txt contains a data table for ZZ Ceti white dwarfs (Cang et al., 2025) with the following columns:

- `Teff(K)` — effective temperature in kelvin;
- `logg` — base-10 logarithm of the surface gravity in cm/s^2;
- `G(mag)` — apparent magnitude in the G band;
- `Parallax(mas)` — parallax in milliarcseconds.

The file BC.txt provides bolometric corrections for the G band (Carrasco et al., 2014):

- `Teff[K]` — effective temperature in kelvin;
- `logg[cm/s2]` — base-10 logarithm of the surface gravity in cm/s^2;
- `Mbol-MG` — bolometric correction.

a) Plot the dependence of the white dwarf radius (in $R_\odot$) on its mass (in $\mathfrak{M}_\odot$).

b) How many objects outlie the main relation? For how many objects are the parameters non-realistic? Explain.

Solution

a) We start from the standard parallax–distance relation: $r/\mathrm{pc} = \dfrac{1}{\varpi/\mathrm{arcsec}}$.

Using this, we can express the absolute magnitude of each object in the G band as

$$M_G = G + 5 - 5\lg(r/\mathrm{pc}) = G + 5 + 5\lg(\varpi/\mathrm{arcsec}) = \texttt{G(mag)} + 5 + 5\lg\frac{\texttt{Parallax(mas)}}{1000\ \mathrm{mas}}.$$

Since most objects are located at distances of only a few hundred parsecs, we can safely neglect the effect of interstellar extinction.

To proceed further, we need the bolometric magnitude. This requires applying a bolometric correction, for which the tabulated values are provided in the file BC.txt. These corrections are given only on a relatively coarse grid of parameters (T_{eff}; $\log g$). Hence, in order to obtain the appropriate value for each individual white dwarf, interpolation across this grid is necessary. In particular, the SciPy library in Python provides convenient tools for that. The corresponding part of the code might look like this:

```
interp = LinearNDInterpolator(
    points=BC_cat[['Teff[K]', 'logg[cm/s2]']].values,
    values=BC_cat['Mbol-MG'].values,
    fill_value=np.nan
)

cat['BC'] = interp(cat[['Teff(K)', 'logg']].values)
```

Alternative way. One might also implement a "manual" bilinear interpolation. Suppose we wish to evaluate $f(x, y)$ at a point $P = (x, y)$ given its values at the four surrounding grid nodes: $Q_{11} = (x_1, y_1)$, $Q_{21} = (x_2, y_1)$, $Q_{12} = (x_1, y_2)$, and $Q_{22} = (x_2, y_2)$.

First, perform linear interpolation along the x-direction to obtain intermediate values at

$$R_1 = (x, y_1), \qquad R_2 = (x, y_2):$$

$$f(R_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}),$$
$$f(R_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}).$$

Next, interpolate in the y-direction between R_1 and R_2:

$$f(P) \approx \frac{y_2 - y}{y_2 - y_1} f(R_1) + \frac{y - y_1}{y_2 - y_1} f(R_2).$$

Altogether this produces the standard bilinear interpolation formula. If P lies outside the rectangle, the same expressions provide a simple linear extrapolation.

After the bolometric corrections are determined, the bolometric magnitude and luminosity of each object can be obtained by comparison with the Sun using Pogson's relation:

$$M_{\text{bol}} = M_G + BC,$$
$$L = L_\odot \, 10^{0.4(M_{\text{bol},\odot} - M_{\text{bol}})}.$$

Next, we apply the Stefan–Boltzmann law. Expressing the stellar radius relative to the Sun yields

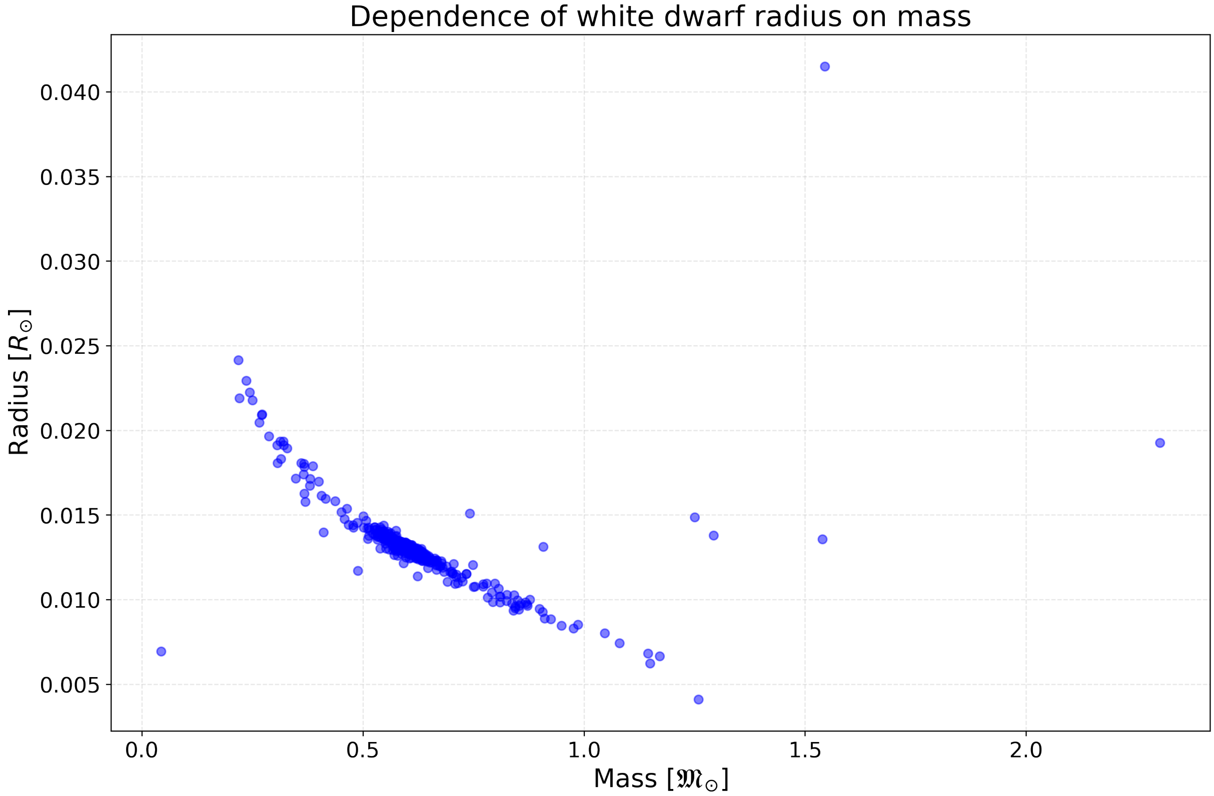
$$L = 4\pi R^2 \sigma T^4 \qquad \Longrightarrow \qquad \frac{R}{R_\odot} = \sqrt{\frac{L}{L_\odot}} \left(\frac{T_\odot}{T}\right)^2.$$

Finally, the mass of each white dwarf follows from the surface gravity and the radius:

$$g = \frac{G\mathfrak{M}}{R^2} \qquad \Longrightarrow \qquad \mathfrak{M} = \frac{gR^2}{G}.$$

Remark. Care must be taken with units: the catalog quantity `logg` $= \log_{10}(g/[\text{cm/s}^2])$, so that the actual surface gravity in SI units is

$$g/\tfrac{\text{m}}{\text{s}^2} = 10^{\texttt{logg} - 2}.$$



b) We identify at least four objects with clearly unrealistic parameters: white dwarf masses cannot exceed the Chandrasekhar limit of about $1.4\,\mathfrak{M}_\odot$, nor can they fall below $\sim 0.1\,\mathfrak{M}_\odot$. In addition, several stars exhibit moderate deviations from the main trend.

Such discrepancies may arise from unaccounted interstellar extinction, the absence of reported observational uncertainties, or hidden binarity of some objects. Nevertheless, the overall distribution agrees reasonably well with the theoretical white dwarf mass–radius relation.

Marking Scheme:

- Question (a) R–$\mathfrak{M}$ relation
  1. Interpolation of the bolometric correction — **5 pt.**
  2. Application of the bolometric correction to the G band magnitude — **1 pt.**
  3. Calculation of absolute magnitudes — **2 pt.**
  4. Calculation of luminosities — **1 pt.**
  5. Calculation of radii — **1 pt.**
  6. Calculation of masses — **2 pt.**
  7. Plot Radius vs. Mass — **4 pt.**

     If Mass is plotted versus Radius, values are not in solar units, or the axes are unlabeled — −1 pt. each.
- Question (b) Outliers
  1. Correct identification of the number of outliers — **2 pt.**
  2. Interpretation of the identified outliers — **2 pt.**

2 At the World's Edge

Cape Fligely ($81^\circ 51' $N; $59^\circ 14'$E) is located on the northern coast of Rudolf Island in Franz Josef Land, Russian Federation. It is the northernmost point of Russia, Europe, and the Eurasian continent as a whole.

a) Find the distance from the cape to the North Pole of the Earth.

b) At what solar declination does the polar day begin at the cape? Assume that atmospheric refraction at the horizon is $35'$.

You are provided with a table eclipses.xlsx of lunar eclipses of the 21st century. The table lists the eclipse date, Saros number, gamma[1], the moments of the Moon's disk contacts with the Earth's shadow (see Fig. 1), and the time of greatest eclipse.

c) How many TOTAL lunar eclipses will occur in the 21st century? PARTIAL? PENUMBRAL?

Point Ξ (0; $59^\circ 14'$E) lies on the equator at the same longitude, somewhere in the Indian Ocean.

d) Determine how many TOTAL lunar eclipses will be visible from both Cape Fligely and point Ξ. List the corresponding event numbers.

Count an eclipse as visible if any part of totality occurs above the horizon. Ignore events visible only in partial phases.

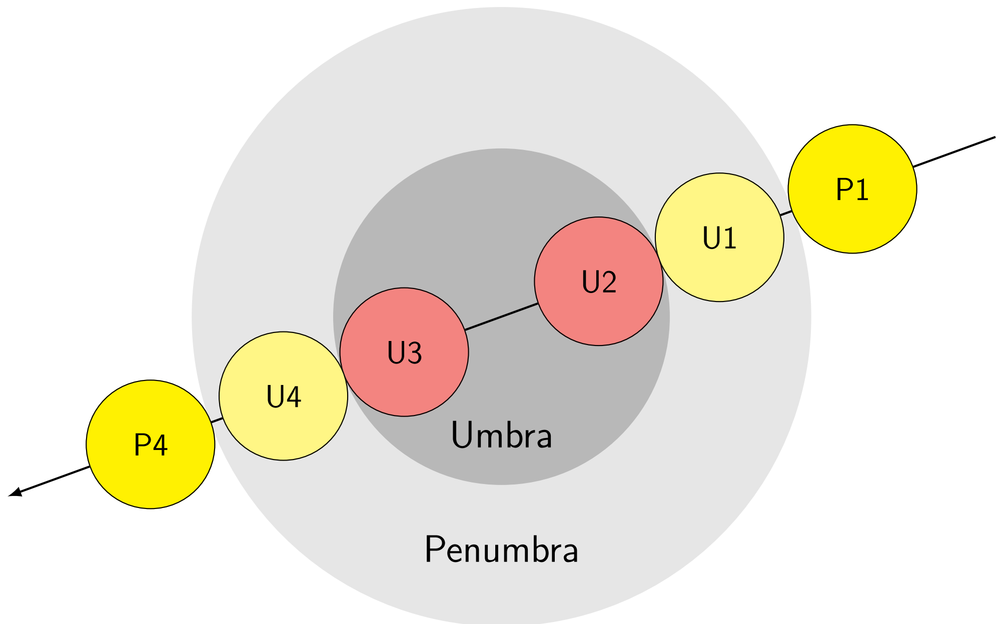


Figure 1: Designation of contacts illustrated with the example of a central total lunar eclipse

[1]Gamma is the distance of the center of the Moon's disk from the center of the Earth's umbral shadow, expressed in units of Earth's equatorial radius. It is defined at the moment of greatest eclipse, when its absolute value reaches a minimum. For a lunar eclipse, it shows whether the Moon passes north or south of the center of the Earth's shadow; a positive value means the Moon passes to the north of the center.

Solution

a) The distance from Cape Fligely to the North Pole is simply the meridional arc:

$$d = \frac{90^\circ - \varphi}{360^\circ} \cdot 2\pi R_\oplus = \frac{90^\circ - 81^\circ\,51'}{360^\circ} \times 2 \times 3.14 \times 6371 \text{ km} \approx 906 \text{ km}.$$

b) The polar day begins when, at lower culmination, the upper limb of the Sun just touches the apparent horizon. In terms of the Sun's center, this condition is

$$h = -\left(\frac{\rho_\odot}{2} + r\right) = -\left(\frac{32'}{2} + 35'\right) = -51' = -0.85^\circ,$$

where $\rho_\odot$ is the Sun's angular diameter and r is the correction for atmospheric refraction. The corresponding solar declination is obtained from the formula for the altitude at lower culmination:

$$\delta_\odot = h - \varphi + 90^\circ = -51' - 81^\circ\,51' + 90^\circ = +7.3^\circ.$$

c) Although the eclipse type can in principle be inferred from the value of γ, a more straightforward approach is to classify it directly from the reported contact times:

- Penumbral: only penumbral contacts, which are not listed in the table;
- Partial: two umbral contacts are present (U1, U4);
- Total: all four umbral contacts are present.

The counts are shown in Table 1.

d) Given the complexity of a fully manual approach, two possible strategies can be pursued:

- to use astronomical software, such as the AstroPy package, to obtain precise results;
- to simplify the problem by considering only the dominant physical effects, thus obtaining an approximate answer with an associated uncertainty.

The code in Appendix A (page 16) demonstrates the first strategy, implemented in Python with AstroPy. Tables 1 and 2 demonstrate the results.

Table 1: Summary statistics of total lunar eclipses in the 21st century

Counts	21st century	Since OWAO 2025
Penubral eclipses	88	66
Partial eclipses	57	44
Total eclipses	85	62
Visible from...		
Cape Fligely	49	36
Point Ξ	47	39
both places	28	23

Table 2: Visibility of total lunar eclipses in the 21st century from Cape Fligely and from point Ξ on the equator (same longitude). A check mark indicates that at least part of totality is visible from the location. The 58th eclipse occured on September 7th, 2025, two weeks before OWAO 2025 started.

Eclipse No.	Visibility: Equator	Cape Fl.	both	Eclipse No.	Visibility: Equator	Cape Fl.	both
1	✓	✓	✓	115	✓		
7				116		✓	
8	✓	✓	✓	117	✓		
9	✓			118	✓	✓	✓
10		✓		123		✓	
15	✓	✓	✓	124			
16				125	✓	✓	✓
17		✓		133	✓		
24		✓		134		✓	
25	✓			140	✓	✓	✓
26	✓	✓	✓	141			
32				142			
33		✓		143	✓	✓	✓
34				148		✓	
35		✓		149	✓		
41		✓		150	✓	✓	✓
42	✓			157		✓	
43		✓		158			
49				159		✓	
51				164	✓	✓	✓
52		✓		165	✓		
57				166		✓	
58	✓	✓	✓	167	✓		
59		✓		174	✓		
66	✓	✓	✓	175		✓	
67				182	✓	✓	✓
68	✓	✓	✓	183			
74	✓			184	✓	✓	✓
75	✓	✓	✓	189	✓	✓	✓
76	✓			190	✓		
77		✓		191	✓	✓	✓
82	✓	✓	✓	199	✓		
83	✓			200		✓	
84	✓	✓	✓	205	✓	✓	✓
92				206	✓	✓	✓
93	✓	✓	✓	207	✓	✓	✓
99	✓			208	✓		
100	✓			215			
101	✓	✓	✓	216	✓	✓	✓
102				224	✓	✓	✓
107	✓	✓	✓	225	✓		
108				226		✓	
109		✓					

Discussion

How can the visibility of a lunar eclipse be assessed without relying on specialized software? The problem reduces to determining whether the Moon is above the local horizon during the total phase. This, in turn, depends on a number of astronomical and observational factors:

- the declination of the Moon, which varies with date;
- the hour angle of the Moon, determined by local time;
- the equation of time, which links apparent solar time to mean time;
- the angular diameter of the Moon;
- atmospheric refraction near the horizon;
- the diurnal parallax of the Moon;
- the eclipse *gamma* parameter, characterizing the Moon's trajectory through the Earth's shadow.

Analysis for Cape Fligely

At Cape Fligely's extreme northern latitude, the Moon's diurnal path is nearly parallel to the horizon. The dominant factor determining visibility is the Moon's declination: a large arc of the ecliptic remains permanently above the horizon, while another part remains permanently below it.

Since a lunar eclipse always occurs at full Moon, when the Moon is in opposition to the Sun, the following conclusions hold:

- Winter eclipses: if the Moon's declination is greater than about $+8^\circ$, the Moon does not set and the eclipse is entirely visible.
- Summer eclipses: if the Moon's declination is less than

$$h_{\min} - \varphi + 90^\circ = -0.85^\circ - 81^\circ 51' + 90^\circ \approx -9^\circ,$$

  then the Moon does not rise and the eclipse is entirely invisible.
- Near the equinoxes: visibility should be checked on a case-by-case basis.

Analysis for point Ξ

At the equator the situation is essentially reversed. The Moon rises and sets every day, spending on average about 12 hours above the horizon (slightly more when minor effects are considered). Consequently, the key factor for visibility is the local time of the eclipse contacts. Since a lunar eclipse occurs at full Moon, the event is visible whenever the Moon is above the horizon, i.e. roughly from six hours before until six hours after local midnight.

The local time at point Ξ is related to UTC by

$$T_\Xi = \mathrm{UTC} + \frac{\lambda}{360^\circ} \times 24^\mathrm{h} = \mathrm{UTC} + \frac{59^\circ\, 14'}{360^\circ} \times 24^\mathrm{h} = \mathrm{UTC} + 3^\mathrm{h}\, 57^\mathrm{m}.$$

Neglecting the equation of time, this means that total eclipses are observable from point Ξ if the totality occurs between about 14:00 and 02:00 UTC. Events close to these boundaries require special attention.

The Moon's topocentric altitude and azimuth at any time and for any terrestrial location may be obtained from authoritative ephemeris services, such as the JPL Horizons system[2].

Marking Scheme:

- Question (a) Distance to the North Pole — **2 pt.**
- Question (b) Polar days — **2 pt.**
- Question (c) Eclipses count
  3+1 pt.: 1 pt. for each correctly obtained amount, 1 pt. for the correct explanation.
- Question (d) Eclipses visibility
  1. Description of the solution methodology — **5 pt.**
  2. Calculation for the Cape — **3 pt.**
  3. Calculation for the point Ξ — **3 pt.**
  4. Obtaining the correct answer — **1 pt.**

Note. To avoid ambiguity, both the total number of eclipses in the 21st century and the number of eclipses since OWAO 2025 (i.e. starting from the 59th event, as the 58th eclipse occurred on September 7th, 2025 shortly before the Olympiad) are accepted as correct, provided the interpretation is stated explicitly.

[2] `https://ssd.jpl.nasa.gov/horizons/app.html`

3 Beyond the World's Edge

Fractal dimension (D) is a fractional value that measures the complexity and roughness of a shape. It describes how a detail-rich structure fills space and often exhibits self-similarity — meaning parts of the structure resemble the whole, at least statistically, when scaled. For example, a perfectly random distribution of points in a plane would have a fractal dimension equal to the dimension of the space itself, $D = 2$. A value less than 2 indicates a "clumpy" structure with significant empty spaces (voids). The transition scale from a fractal to a homogeneous universe is an active area of research.

The file `SN_cat.csv` provides observational data for supernova events. The columns are:

- `sn_name` supernova designation;
- `redshift` measured redshift;
- `gal_type` morphological type of the host galaxy;
- `sn_ra` right ascension in hours, minutes, and seconds;
- `sn_dec` declination in degrees, minutes, and seconds.

a) Plot the objects on a rectangular sky map with axes (Right Ascension; Declination) in degrees. Are there any regions without objects? What does the largest such region represent?

For the remainder of the problem, ***we will only consider a region of the sky***

$$\delta \in [-5^\circ;\ +5^\circ],$$
$$\alpha \in [300^\circ;\ 360^\circ] \cup [0^\circ;\ 60^\circ],$$

where the object sample seems to be complete.

b) For the selected region, plot a histogram of the redshift distribution.

Keep only objects with redshift $z \leqslant 0.1$.

For these objects, we will determine the fractal dimension. To do this, it is necessary to calculate the pairwise distances $l_{i,j}$ for all pairs (i, j) of the remaining objects. The redshift is small — use ordinary Euclidean distance without applying cosmological models.

c) Plot a histogram for the distribution $f(l)$ of pairwise distance values.

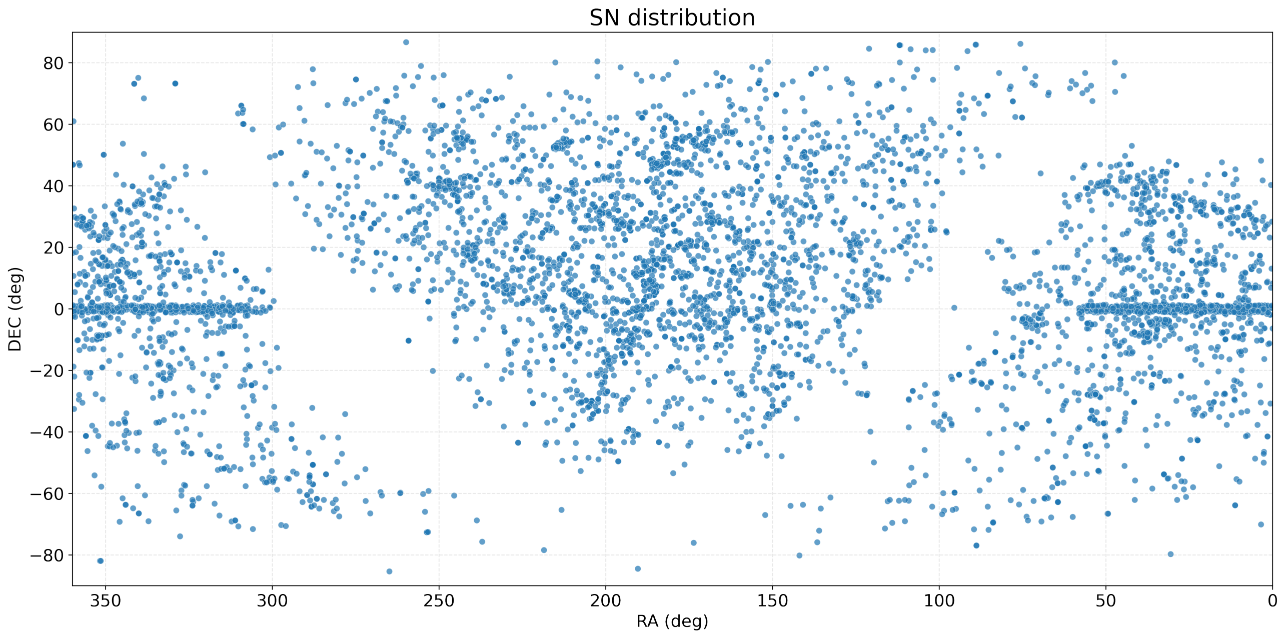
d) To determine the fractal dimension, use pairwise distances l in the range from 1 Mpc to 100 Mpc. Fit the distribution with a power law $f(l) = A\,l^D$.

The sample objects populate an oblate, practically 2D-region. Therefore, D is the fractal dimension of the two-dimensional projection. We may use Mandelbrot's empirical rule to estimate the fractal dimension D_0 of the spatial (3D) distribution of supernovae: $D_0 = D + 1$.

e) Write down the obtained value of D_0. Is D_0 greater or less than 3?

Solution

a) We first convert the right ascension from hours to degrees (using the factor 15° per hour), and then plot the supernovae on a rectangular sky map. As customary in astronomy, right ascension increases from right to left.



A broad V-shaped band with almost no objects is clearly visible. This feature corresponds to the projection of the Galactic plane onto the celestial sphere. Because the Galactic disk contains large amounts of gas and dust, light from extragalactic supernovae is absorbed, and distant sources in this region remain unobservable.

b) The selected sky region contains a total of 1090 supernovae with redshifts in the range $0.00385 \leqslant z \leqslant 1.23000$. The corresponding distribution (Fig. 2) shows a sharp decline in the number of detections for $z \gtrsim 0.4$.

c) After applying the redshift cut ($z \leqslant 0.1$), the sample is reduced to 329 objects. Although this number may seem modest, it already yields $329 \times 328/2 = 53\,956$ pairwise distances for statistical analysis.

At such low redshifts, distances are estimated from the velocitydistance relation:

$$d = \frac{v}{H_0}, \qquad v = cz.$$

The pairwise separations $l_{i,j}$ are then obtained using Euclidean geometry. The angular separation $\theta_{i,j}$ follows from the spherical law of cosines:

$$\cos\theta_{i,j} = \sin\delta_i \sin\delta_j + \cos\delta_i \cos\delta_j \cos(\alpha_i - \alpha_j).$$

Finally, the spatial separation is given by the planar law of cosines:

$$l_{i,j} = \sqrt{d_i^2 + d_j^2 - 2d_i d_j \cos\theta_{i,j}}.$$

The resulting distribution (Fig. 3) rises up to ~ 50 Mpc and declines thereafter. This behavior reflects both the finite size of the sample and observational selection effects.

d) Within the range $1 \leqslant l \leqslant 100$ Mpc we find 13 784 pairwise distances. To estimate the fractal dimension we fit their distribution with a power law. It is convenient to work in logarithmic form:

$$\log_{10} f(l) = D \cdot \log_{10} l + \log_{10} A.$$

The coefficients can be obtained by least-squares regression applied to the binned histogram values. We adopt 100 bins, which is consistent with the heuristic choice $\sim \sqrt{N}$ for $N \approx 13\,784$. The resulting distribution on logarithmic axes is shown in Figure 4, together with the best-fitting straight line.

Important remark. When constructing a histogram for logarithmic analysis, the binning *must be uniform* in $\log l$, and logarithm of bin counts is taken. Applying logarithms to counts from linearly spaced bins produces a biased slope, since the data points are not uniformly distributed along the x-axis. Figure 5 illustrates this incorrect approach.

From the regression we obtain $D = 1.61 \approx 1.6$. Using Mandelbrots empirical rule, the inferred fractal dimension of the three-dimensional distribution is

$$D_0 = D + 1 = 2.6 < 3,$$

which indicates that the supernova distribution remains structured rather than homogeneous on these scales.

Note 1. The algorithm employed in this problem follows the approach developed by V. Orlov, A. Raikov, and R. Gerasim[3].

Note 2. The fractal dimension of the Universe is not fixed, but depends on the observational scale. On relatively small scales ($\lesssim 20$ Mpc), the distribution is highly inhomogeneous: galaxies cluster into groups and small clusters, which in turn align along filaments surrounding large empty voids. The resulting "cosmic web" is clearly visible. Here the fractal dimension is well below 3, reflecting the spongy, network-like nature of the structure.

[3]See, for example, Raikov, A. A., Orlov, V. V. & Gerasim, R. V. Determination of the Fractal Dimensionality of Large-Scale Structure With Type Ia Supernovae by the Method of Pairwise Distances. *Astrophysics* **57,** 287–295 (2014). `https://doi.org/10.1007/s10511-014-9334-9`.

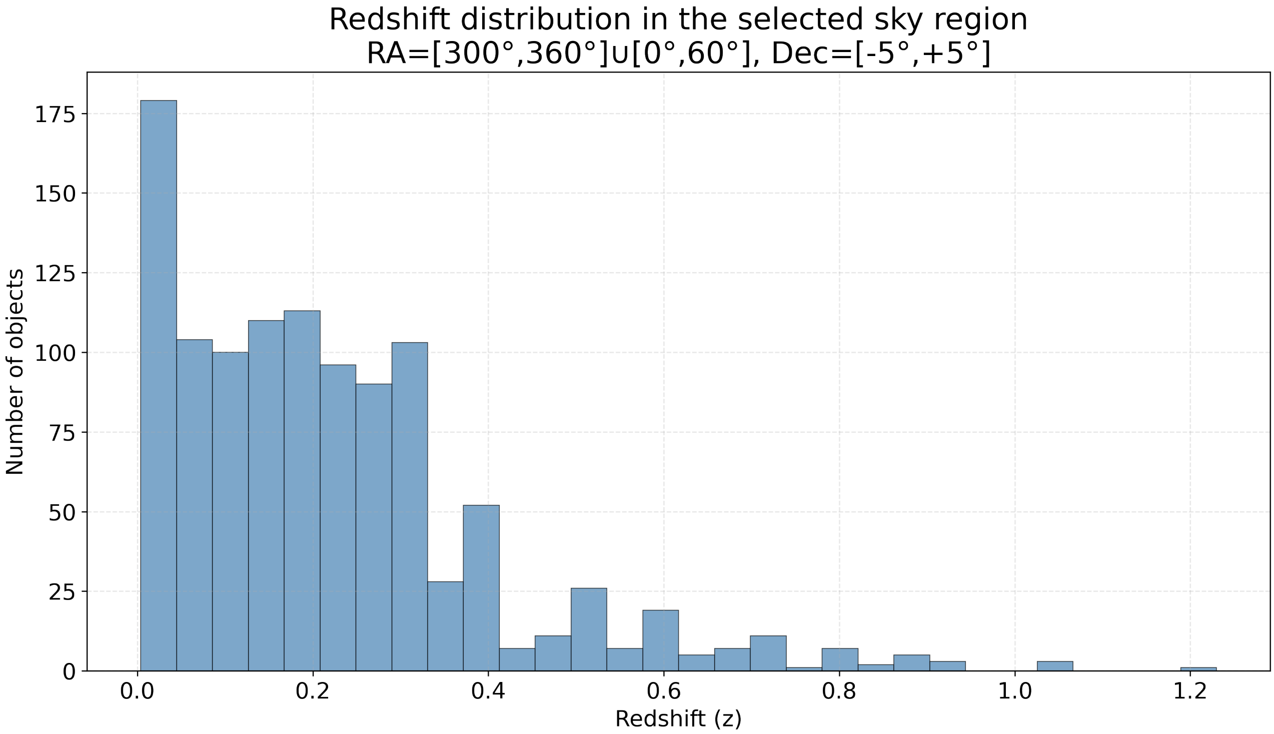


Figure 2: Redshift distribution of the objects

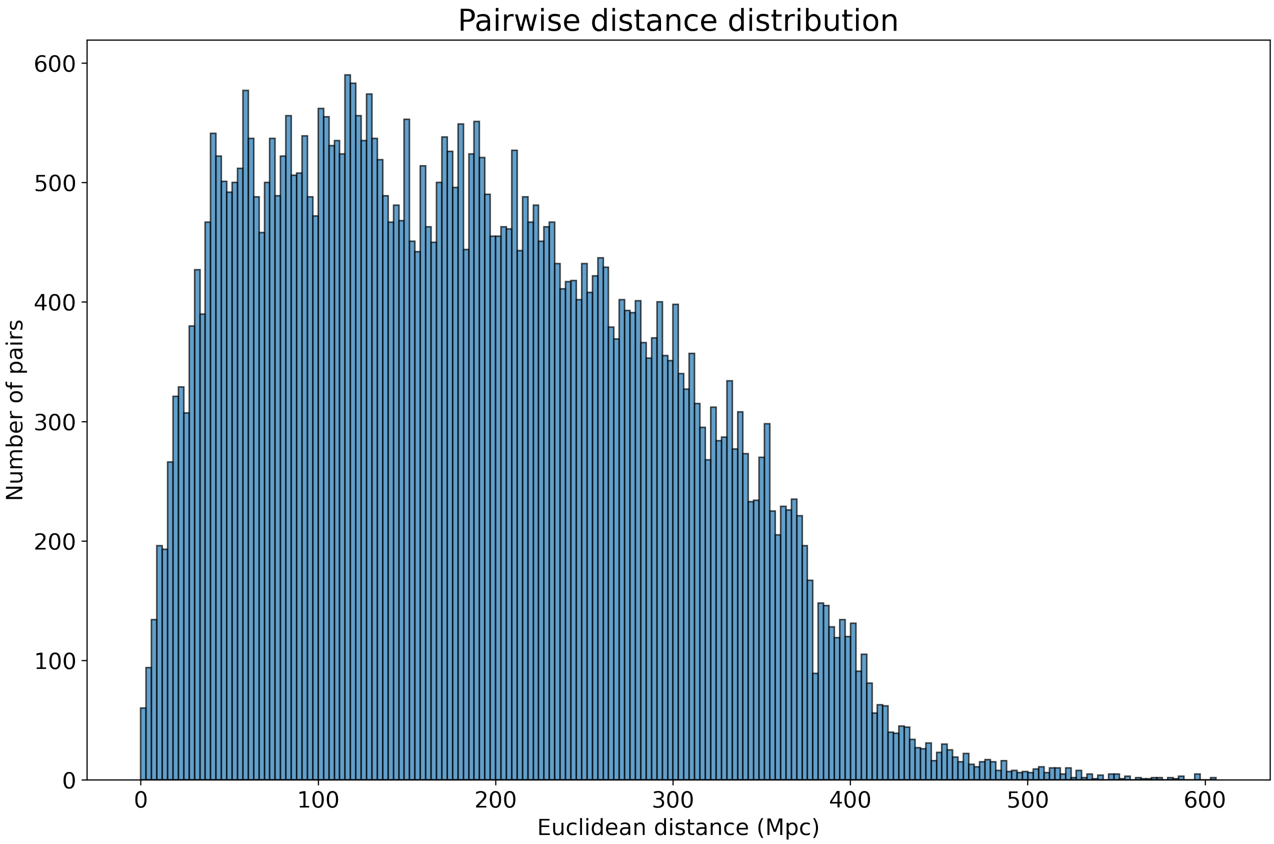


Figure 3: Distance distribution of the objects

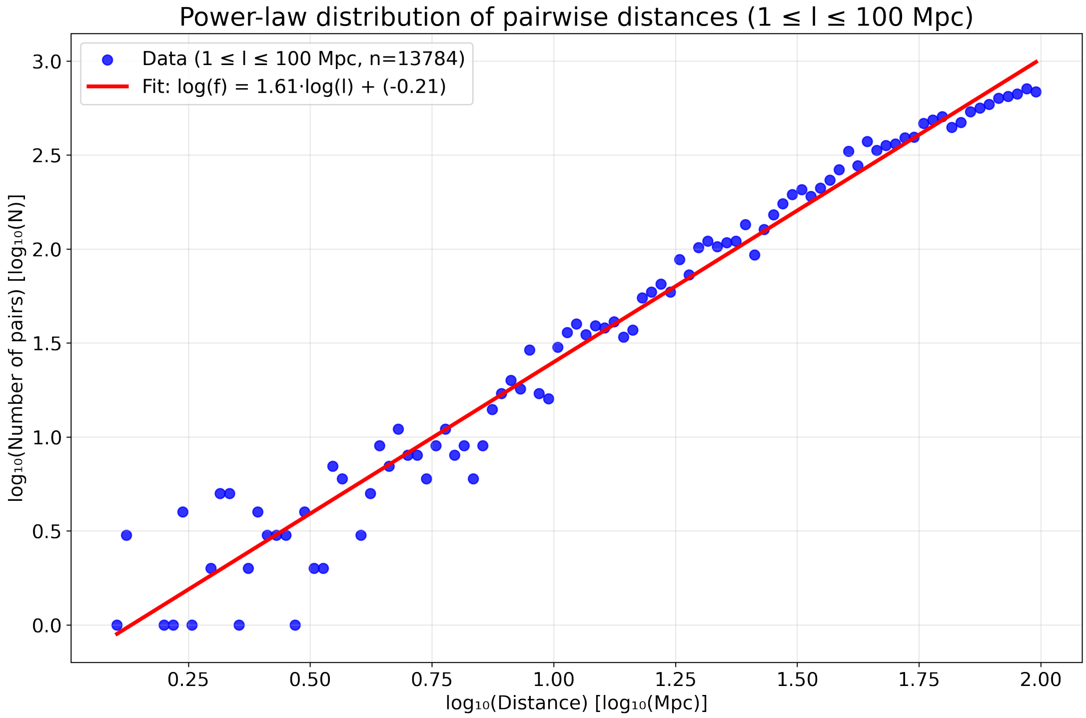


Figure 4: Linear approximation of the pairwise distance distribution function in logarithmic axes

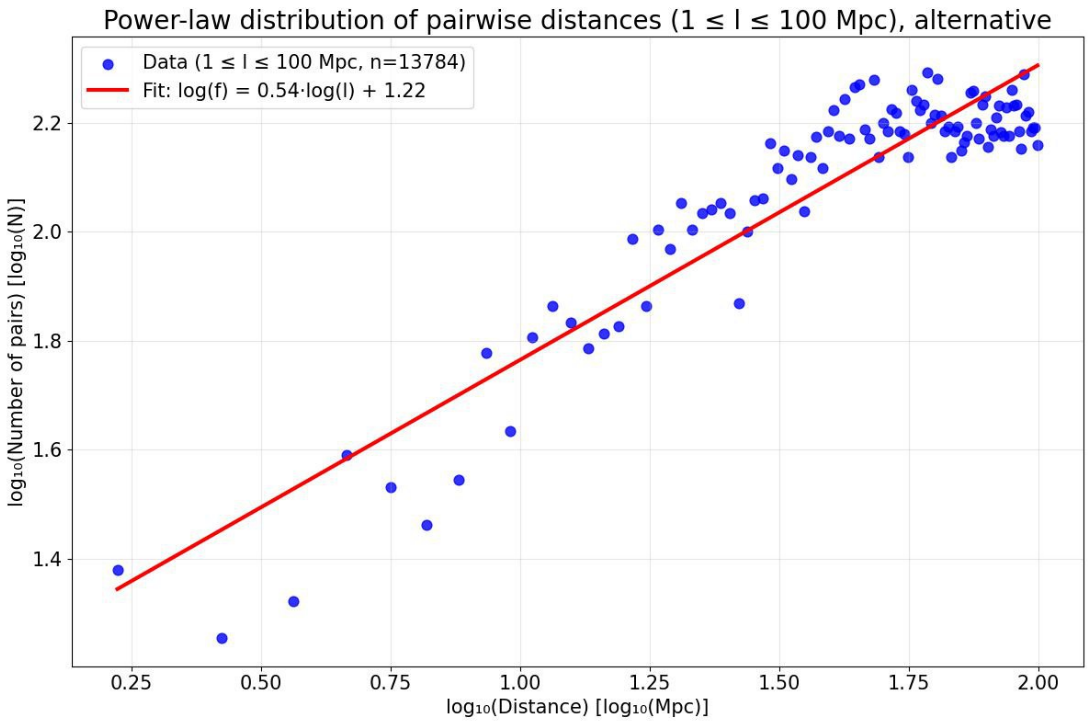


Figure 5: Incorrect linear approximation of the pairwise distance distribution function in logarithmic axes

In the intermediate range (20–100 Mpc), we encounter the transition or cross-over scale. As the sampling volume grows, individual filaments and voids are averaged over, the enclosed number of galaxies increases more rapidly, and the fractal dimension rises toward the homogeneous value of 3.

On the largest observable scales ($\gtrsim$ 300 Mpc), the Universe becomes statistically homogeneous. While structure is still present, any sufficiently large region contains approximately the same number of galaxies as any other region of equal size. The effective fractal dimension thus approaches $D = 3$, indicating that matter fills space uniformly when averaged over such vast scales.

Marking Scheme:

- Question (a) Distribution on the celestial sphere
  1. Correct processing of the initial data — **2 pt.**
  2. Constructing the supernova map — **2 pt.**

     Here and elsewhere, "constructing" implies obtaining the final image; providing the code that generates it is not sufficient.
  3. Detecting and correctly explaining the void region — **2 pt.**
- Question (b) Redshift distribution
  1. Correct data processing — **3 pt.**
  2. Constructing the redshift distribution — **2 pt.**
- Question (c) Pairwise distances distribution
  1. Correct calculation of the pairwise distances — **3 pt.**
  2. Constructing the distribution of pairwise distances — **2 pt.**
- Question (d) Power law fitting — **2 pt.**
- Question (e) Result for fractal dimension — **2 pt.**

  Points are awarded if a correct and well-founded estimate of D_0 was obtained.

Constants

Universal

Gravitational constant $G = 6.67 \cdot 10^{-11}\ \frac{m^3}{kg \cdot s^2}$

Speed of light $c = 3.00 \cdot 10^8$ m/s

Astronomical

Astronomical unit $1\ \text{au} = 149.6 \cdot 10^6$ km

Parsec $1\ \text{pc} = 206\,265$ au

Hubble constant $H_0 = 70$ (km/s)/Mpc

Emission constants

Stefan–Boltzmann $\sigma = 5.67 \cdot 10^{-8}\ \frac{W}{m^2 \cdot K^4}$

Wien's displacement $b = 2898$ µm · K

Earth

Radius $R_\oplus = 6371$ km

Obliquity of ecliptic $\varepsilon = 23.4°$

Orbital period $T_\oplus = 365.26$ days

Sun

Radius $R_\odot = 6.96 \cdot 10^5$ km

Mass $\mathfrak{M}_\odot = 1.99 \cdot 10^{30}$ kg

Absolute magnitude $M_\odot = 4.74^m$ (bol.)

Effective temperature $T_\odot = 5.8 \cdot 10^3$ K

Luminosity $L_\odot = 3.828 \cdot 10^{26}$ W

A Analysis of Eclipse Visibility with AstroPy

```python
"""
Analysis of Lunar Eclipse Visibility
====================================

This script reads a catalog of 21st century lunar eclipses from
`eclipses.xlsx`, extracts the times of totality, and determines
which eclipses are visible from:
    - Cape Fligely
    - Point Xi on the equator

Visibility is defined as the Moon's upper limb being above the
geometric horizon, corrected for refraction and the Moon's
radius.
"""

import pandas as pd
import numpy as np
import astropy.units as u
from astropy.time import Time, TimeDelta
from astropy.coordinates import EarthLocation, AltAz, get_body


# ---------------------------------------------------------------------
# Load and preprocess the eclipse catalog
# ---------------------------------------------------------------------
def load_eclipses(path: str = "eclipses.xlsx") -> pd.DataFrame:
    """
    Load the lunar eclipse catalog from Excel and standardize columns.

    Parameters
    ----------
    path : str
        Path to the Excel file.

    Returns
    -------
    df : pandas.DataFrame
        DataFrame with standardized columns and auxiliary fields.
    """
    df = pd.read_excel(path, skiprows=1)

    df = df.rename(columns={
        "Unnamed: 0": "num",
        "U1": "U1", "U2": "U2", "U3": "U3", "U4": "U4",
        "Unnamed: 10": "maxt"
    })

```

```
    months = {m: i for i, m in enumerate(
        ["January","February","March","April","May","June",
         "July","August","September","October","November","December"], 1)}
    df["Month_num"] = df["Month"].map(months)

    df["date"] = pd.to_datetime(
        dict(year=df["Year"], month=df["Month_num"], day=df["Day"]),
        errors="coerce"
    )

    def frac_to_time(frac):
        if pd.isna(frac):
            return np.nan
        seconds = float(frac) * 24 * 3600
        h = int(seconds // 3600)
        m = int((seconds % 3600) // 60)
        s = int(seconds % 60)
        return f"{h:02d}:{m:02d}:{s:02d}"

    df["maxt_str"] = df["maxt"].apply(frac_to_time)

    return df


# -----------------------------------------------------------------------
# Convert tabulated times into astropy.Time arrays
# -----------------------------------------------------------------------
def build_times(df: pd.DataFrame):
    """
    Construct astropy.Time objects for total eclipses only.

    Returns
    -------
    df_tot : pandas.DataFrame
        Subset containing only total eclipses (U2 and U3 defined).
    t2, t3, tmax : astropy.Time
        Times of second contact, third contact, and maximum eclipse.
    """
    df_tot = df[df["U2"].notna() & df["U3"].notna()].copy()

    def to_iso(dates, times):
        iso_list = []
        for d, t in zip(dates, times):
            if pd.isna(t) or pd.isna(d):
                iso_list.append("NaT")
            else:
                iso_list.append(f"{d.strftime('%Y-%m-%d')}T{t}")
        return np.array(iso_list, dtype=str)

    iso_u2  = to_iso(df_tot["date"], df_tot["U2"])
```

```
    iso_u3  = to_iso(df_tot["date"], df_tot["U3"])
    iso_max = to_iso(df_tot["date"], df_tot["maxt_str"])

    t2   = Time(iso_u2,  format="isot", scale="utc", out_subfmt="date_hms")
    t3   = Time(iso_u3,  format="isot", scale="utc", out_subfmt="date_hms")
    tmax = Time(iso_max, format="isot", scale="utc", out_subfmt="date_hms")

    # adjust for midnight crossings
    one_day = TimeDelta(1*u.day)
    t3[tmax > t3] = t3[tmax > t3] + one_day
    t2[tmax < t2] = t2[tmax < t2] - one_day

    return df_tot, t2, t3, tmax


# ----------------------------------------------------------------------
# Visibility criterion
# ----------------------------------------------------------------------
def visible_mask_for_site(t2, t3, lat_deg, lon_deg=59 + 14/60, height_m=0,
                          n_samples=11, h_thresh_deg=-(35+16)/60.0):
    """
    Check eclipse visibility over the full totality interval [U2, U3]
    Samples each eclipse at `n_samples` evenly spaced times
    and returns True if the Moons center altitude exceeds `h_thresh_deg`

    Parameters
    ----------
    t2, t3 : astropy.Time
        Start and end of totality.
    lat_deg, lon_deg : float
        Geographic coordinates of the site (degrees).
    height_m : float
        Elevation above sea level.
    n_samples : int, optional
        Number of samples per eclipse (inclusive of endpoints), default 11.
    h_thresh_deg : float, optional
        Altitude threshold (degrees) for the Moons center.

    Returns
    -------
    mask : ndarray of bool
        True if any sample during totality is above the threshold
        (i.e., visible).
    """
    loc = EarthLocation(
        lat=lat_deg*u.deg,
        lon=lon_deg*u.deg,
        height=height_m*u.m
    )

```

```python
    frac = np.linspace(0.0, 1.0, n_samples)
    dt = t3 - t2
    t_grid = t2[:, None] + frac[None, :] * dt[:, None]

    alt = get_body("moon", t_grid.reshape(-1), location=loc)\
        .transform_to(AltAz(obstime=t_grid.reshape(-1), location=loc))\
        .alt.deg.reshape(len(t2), n_samples)

    return (alt > h_thresh_deg).any(axis=1)


# ------------------------------------------------------------------------
# Main pipeline
# ------------------------------------------------------------------------
def analyze_visibility(path="eclipses.xlsx"):
    """
    Analyze visibility of total lunar eclipses from Cape Fligely
    and point Xi.
    """
    df = load_eclipses(path)
    df_tot, t2, t3, tmax = build_times(df)

    events = df_tot["num"].to_numpy()

    # two sites
    vis_XI = visible_mask_for_site(t2, t3, lat_deg=0.0)
    vis_FL = visible_mask_for_site(t2, t3, lat_deg=81 + 51/60.0)

    xi_list  = events[vis_XI].tolist()
    fl_list  = events[vis_FL].tolist()
    both     = sorted(set(xi_list) & set(fl_list))

    print("Total eclipses:", len(events))
    print("Visible from point Xi:", len(xi_list))
    print("Visible from Cape Fligely:", len(fl_list))
    print("Visible from both:", len(both))

    return xi_list, fl_list, both


# ------------------------------------------------------------------------
# Entry point
# ------------------------------------------------------------------------
if __name__ == "__main__":
    xi_list, fl_list, both = analyze_visibility("eclipses.xlsx")
```