1 A Plot Twist in Every Day

The file eopc04.txt gives daily measurements from the International Earth Rotation and Reference Systems Service (IERS) at the Paris Observatory. It records high-precision daily measurements of Earth's motion and rotation:

- `Date, MJD` — the observation date and the corresponding Modified Julian Date (MJD);
- `x, y` — the position of Earth's North Pole in two orthogonal coordinate directions, in arcseconds;
- `LOD` — Length of Day, the difference, in seconds, between the measured day length and 86 400 seconds. Positive LOD means a longer day.

The Pole's Path

a) Plot the pole's path in the xy-plane. Use the same scale on both axes (an *equal aspect ratio*) so the path keeps its true shape.

b) Find the pole's greatest distance from the origin $(0, 0)$. Give this distance in arcseconds and in meters.

In which year did the pole reach this maximum distance?

The Hidden Period

c) Plot the pole's distance r from the origin against time. Do not smooth the data.

d) The plot shows a complex pattern made by adding two periodic signals. One signal has a period of $T_1 = 365.25$ days. Estimate the second period, T_2.

LOD Trends and Seasonal Changes

e) Convert the LOD values to milliseconds, then plot LOD against time.

f) Fit a straight line $y = kx + b$ to the data, where x is the year and y is LOD in milliseconds. Find the slope k, and express it in milliseconds per century. Compare your result with the known estimate that tidal friction caused by the Moon slows Earth's rotation by about 1.7 milliseconds per century.

g) Subtract the fitted trend from the LOD values in milliseconds to remove the long-term change. Group the residuals by calendar day number (from 1 to 365) and find the average for each day number across all years. Plot the average LOD residual against calendar day number.

h) In which Northern Hemisphere season — winter or summer — are the days longer? Suggest how seasonal changes in Earth's atmosphere might cause this pattern.

2 Compact Choir

The file GWs.txt lists mergers of compact objects:

- `Name` — the event name;
- `m1_det` and `m2_det` — the two component masses measured in the detector frame, in $\mathfrak{M}_\odot$;
- `D_L` — the luminosity distance to the event, in megaparsecs (Mpc);
- `z*` — the redshift z.

Assume a flat ΛCDM universe, so $\Omega_\Lambda = 1 - \Omega_m$, and the luminosity distance is

$$D_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{d\tilde{z}}{\sqrt{\Omega_m(1+\tilde{z})^3 + \Omega_\Lambda}}.$$

Find the Matter Density

a) The only unknown parameter in the model is Ω_m. Find its best-fit value.

b) Plot the measured luminosity distance D_L against redshift z. Add the theoretical curve using your best-fit value of Ω_m.

Object Masses and Types

A *source-frame mass* is the mass of an object in the rest frame of the binary system. A *detector-frame mass* is the mass inferred from the gravitational-wave signal measured by a detector on Earth, before correcting for redshift. Cosmic expansion redshifts the signal, so the masses are related by

$$m_{\text{det}} = (1+z)m_{\text{src}}.$$

For this task, classify objects with source-frame masses $m \leqslant 2.3\mathfrak{M}_\odot$ as neutron stars (NS), objects with $m \geqslant 3.0\mathfrak{M}_\odot$ as black holes (BH), and objects with $2.3\mathfrak{M}_\odot < m < 3.0\mathfrak{M}_\odot$ as *Uncertain*.

c) For each event, find the total source-frame mass $m_{\text{src}} = m_{1,\text{src}} + m_{2,\text{src}}$. Plot it against redshift z. Describe the main trend. Explain why the plot has this shape.

d) Make a histogram of source-frame component masses. Is there a *mass gap* (a range with few or no objects) between neutron stars and black holes? Justify your answer.

e) Count the following:

- the number of NS, Uncertain, and BH components;
- the number of events with at least one NS;
- the number of events with at least one Uncertain component;
- the number of events in which both objects are BHs.

f) For each event, calculate the mass ratio

$$q = \frac{\min(m_1, m_2)}{\max(m_1, m_2)}.$$

Make a histogram of the q values. What is the median value of q?

g) At merger, the objects emit gravitational waves with a characteristic frequency

$$f_{\text{merger}} \approx \frac{c^3}{G\pi m_{\text{src}}} \cdot \frac{1}{6\sqrt{6}}.$$

Plot frequency against source-frame mass using logarithmic scales on both axes.

What is the frequency range of these events, from the lowest to the highest frequency? Compare it with LIGO's approximate sensitivity range of 10 to 1000 Hz.

3 A Comet's Visit

On 17 January 2016, comet C/2013 US10 (Catalina) was close to its nearest point to Earth. Its horizontal parallax was $\approx 12''$. On 18 March 2016, the horizontal parallax was $\approx 4.0''$.

A simple estimate

a) Use the horizontal parallax values to find the distance from Earth to the comet on 17 January and on 18 March. Give your answers in astronomical units.

b) Assume that the comet moved in a *straight line* relative to Earth. Take 17 January as the moment of closest approach. Estimate the relative distance it travelled in 61 days and its average relative speed in km/s.

Keplerian motion

The table below gives approximate starting positions and velocities on 17 January 2016 at 06:00 UTC. The Sun is at the origin, the coordinate axes are fixed, and the xy plane is the ecliptic plane.

	Earth	**Comet**
x (AU)	−0.43622849	−0.80691474
y (AU)	+0.88172514	+0.94496430
z (AU)	−0.00003220	+0.61939535
v_x (AU/day)	−0.01570046	+0.00524790
v_y (AU/day)	−0.00769377	+0.01726800
v_z (AU/day)	+0.00000028	+0.01002658

c) Calculate the positions of Earth and the comet for the next 61 days considering Keplerian motion.

d) For each day, find the comet's position relative to Earth. Plot the comet's relative path in the ecliptic plane.

e) Find the total length of the comet's path relative to Earth during these 61 days. Calculate its average relative speed in km/s.

How curved was the relative path?

f) Compare the average path speed from part (e) with the simple estimate from part (b). Find the percentage difference.

g) Make a second model in which the comet moves relative to Earth in a straight line, keeping its initial relative velocity. On the same graphs, plot the actual (Keplerian-model) relative path and the straight-line path in the xy and xz planes.

h) Find the distance between the two predicted positions of the comet after 61 days in AU.

i) Based on your results, which quantity is better estimated by the second straight-line model defined in part (g): the average relative speed or the comet's relative position after 61 days? Justify your answer.

Constants

Universal

Gravitational constant	$G = 6.67 \cdot 10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg \cdot s}^2}$
Speed of light	$c = 3.00 \cdot 10^8\ \mathrm{m/s}$

Astronomical

Astronomical unit	$1\ \mathrm{au} = 149.6 \cdot 10^6\ \mathrm{km}$
Parsec	$1\ \mathrm{pc} = 206\,265\ \mathrm{au}$
Hubble constant	$H_0 = 67.4\ \mathrm{(km/s)/Mpc}$

Emission constants

Stefan–Boltzmann	$\sigma = 5.67 \cdot 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4}$
Wien's displacement	$b = 2898\ \mu\mathrm{m} \cdot \mathrm{K}$

Earth

Radius	$R_\oplus = 6371\ \mathrm{km}$
Obliquity of ecliptic	$\varepsilon = 23.4^\circ$
Orbital period	$T_\oplus = 365.26\ \mathrm{days}$

Sun

Radius	$R_\odot = 6.96 \cdot 10^5\ \mathrm{km}$
Mass	$\mathfrak{M}_\odot = 1.99 \cdot 10^{30}\ \mathrm{kg}$
Absolute magnitude	$M_\odot = 4.74^\mathrm{m}$ (bol.)
Effective temperature	$T_\odot = 5.8 \cdot 10^3\ \mathrm{K}$
Luminosity	$L_\odot = 3.828 \cdot 10^{26}\ \mathrm{W}$